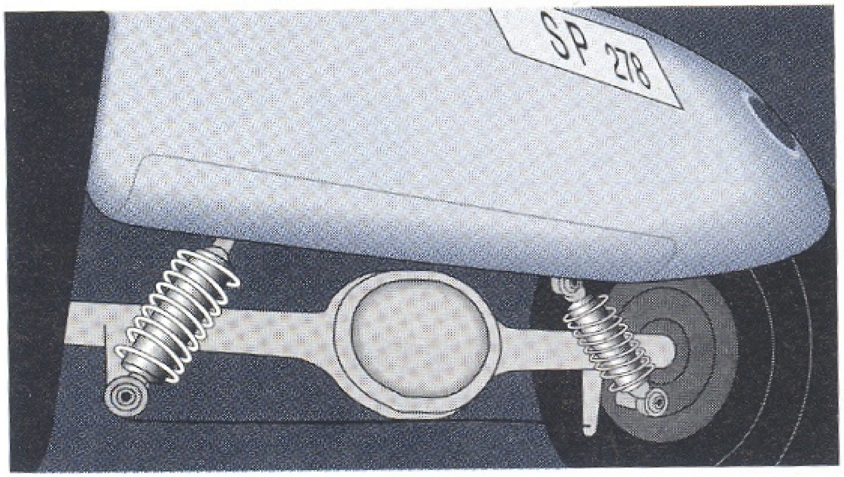
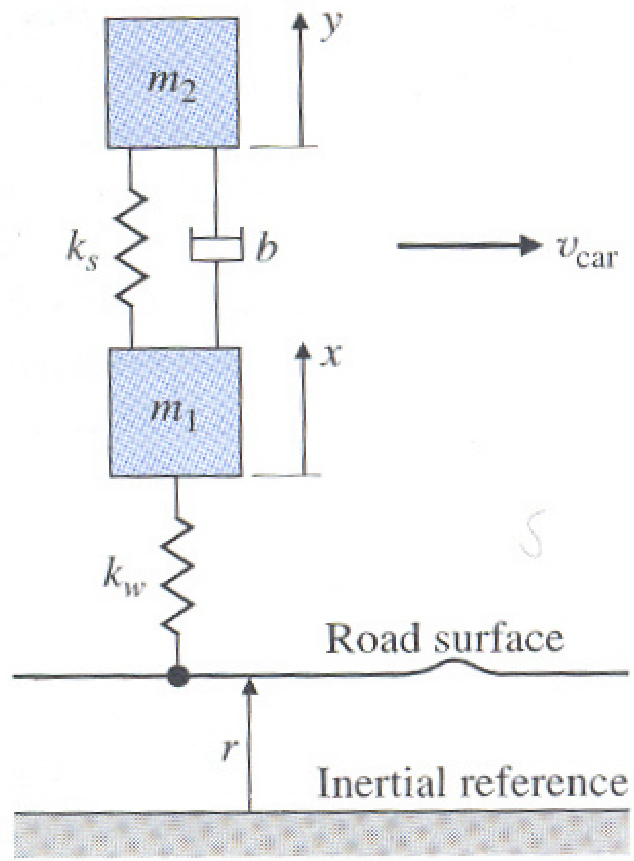
➢ How derive mathematical models of systems ?

$$b(\dot{y}-\dot{x})+k_s(y-x)-k_w(x-r)=m_1\ddot{x},$$

$$-k_s(y-x)-b(\dot{y}-\dot{x})=m_2\ddot{y}$$





Suspension System of Car

$$\ddot{x}+\frac{b}{m_1}(\dot{x}-\dot{y})+\frac{k_s}{m_1}(x-y)+\frac{k_w}{m_1}x=\frac{k_w}{m_1}r$$

$$\ddot{y}+\frac{b}{m_2}(\dot{y}-\dot{x})+\frac{k_s}{m_2}(y-x)=0$$

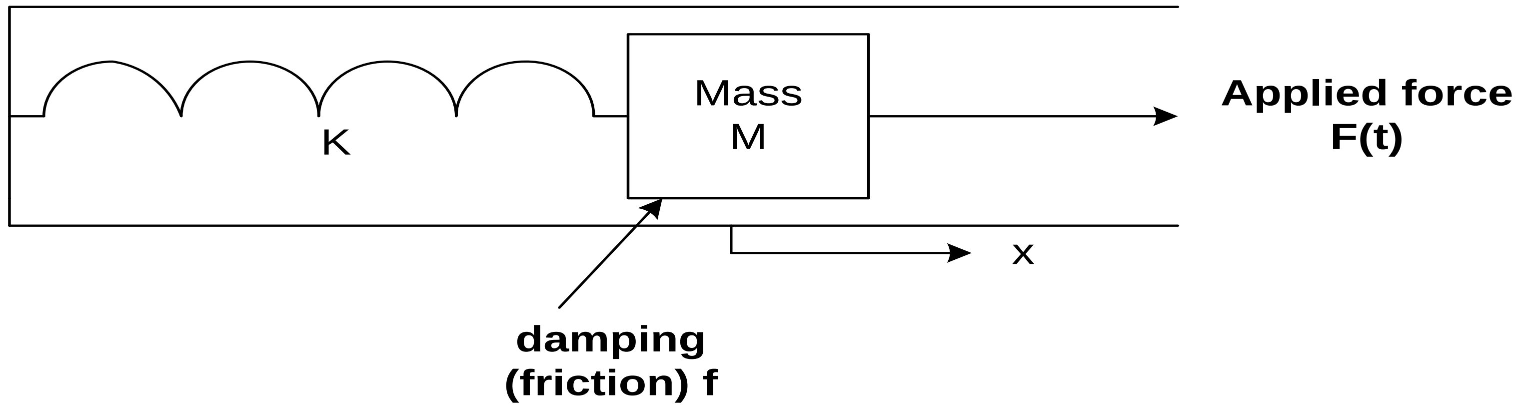
Mathematical Models of Systems

The modeling procedure is summarized as follows:

1. **Define the system and its components.**
2. **List all assumptions.**
3. **Formulate the mathematical model.**
4. **Write the differential (Laplace transform, state-space) equations describing the model.**
5. **Solve the equations for the desired output variables.**
6. **Examine the solutions (often times by comparing with experimental results).**
7. **If necessary, revise assumptions and model, then analyze again.**

Modeling: A Damped Spring Mass System

Example: consider the pictured damped spring mass system



1. The system consists of a mass M and a spring with spring constant K.

2. Assume

(1) The spring obeys Hookes' Law: i.e. restoring force is proportional to displacement from equilibrium position.

(2) The damping is proportional to the speed of the mass M as it moves along the surface.

3,4. Write the describing equation: by Newton's second law

$$F_{net} = Ma = M\frac{d^2x(t)}{dt^2}$$

$Now,$

$$F_{net} = F(t) - Kx(t) - f\frac{dx(t)}{dt}$$

$F(t): applied\ force, Kx(t): restoring\ force\ of\ spring$

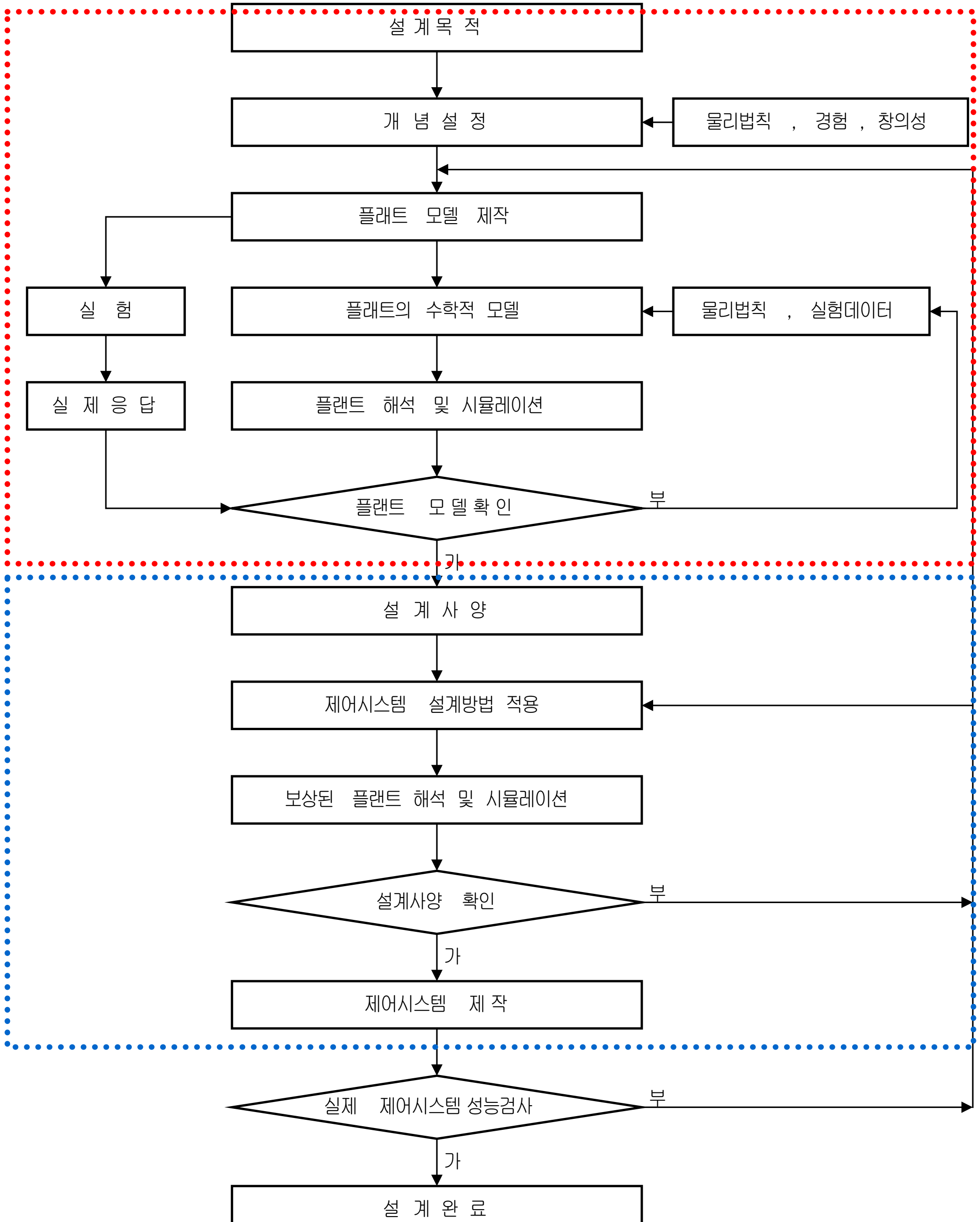
$f\frac{dx(t)}{dt}: damping\ force$

$so\ the\ differential\ equation\ is$

$$M\frac{d^2x(t)}{dt^2} + f\frac{dx(t)}{dt} + Kx(t) = F(t)$$

5,6,7. Solve the equations for the desired outputs : Examine the solutions

제어시스템의 설계 절차



설 계 목 적
개 념 설 정
물리법칙 , 경험 , 창의성
플래트 모델 제작
실 험
플래트의 수학적 모델
물리법칙 , 실험데이터
실 제 응 답
플랜트 해석 및 시뮬레이션
플랜트 모 델 확 인
부
가
설 계 사 양
제어시스템 설계방법 적용
보상된 플랜트 해석 및 시뮬레이션
설계사양 확인
부
가
제어시스템 제 작
실제 제어시스템 성능검사
부
가
설 계 완 료

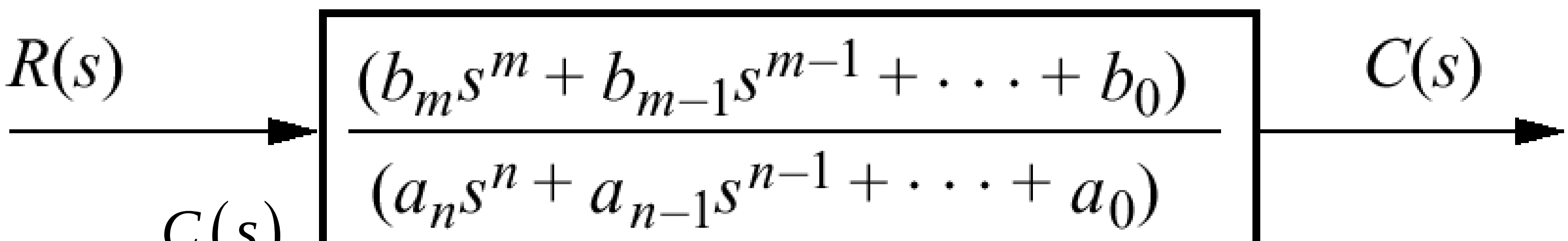
●Transfer Function(전달함수)

선형 시불변(linear time- invariant) 시스템 => 주파수역에서 해석

전달함수 G(s)는 모든 초기조건을 0으로 가정

(∵ 정상상태의 응답 및 동특성을 다루기 때문)

$$G(s)=\frac{y(s)}{u(s)}$$



$$G(s)=\frac{C(s)}{R(s)}$$

● 일반적인 선형 시불변 시스템에 대한 전달함수

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 \dot{y} + a_0 y = b_m u^{(m)} + b_{m-1} u^{(m-1)} + \cdots + b_1 \dot{u} + b_0 u \quad (n \geq m) \tag{2.51}$$

초기조건을 0으로 가정하고 Laplace 변환하면

$$\left(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0\right) y(s) = \left(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0\right) u(s) \tag{2.52}$$

$$G(s) = \frac{y(s)}{u(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} = \frac{N(s)}{D(s)} \tag{2.53}$$

$D(s)$: 특성다항식, $D(s)=0$: 특성방정식을 만족하는 s의 값: 극점
시스템의 안정도와 성능에 영향을 줌

$N(s)$: 영점다항식, $N(s)=0$ 을 만족하는 s의값 :영점
상대안정도와 과도응답에 중요한 역할을 함

전달함수: 선형 시불변 시스템의 입-출력 관계를 나타냄.
물리적으로 서로 다른 시스템에 대한 전달함수가 동일할 수 있다.

● 시스템의 극점 및 영점을 복소 s-평면에 표시

$$G(s) = \frac{s+3}{(s+1)(s+2)} \qquad (2.54)$$

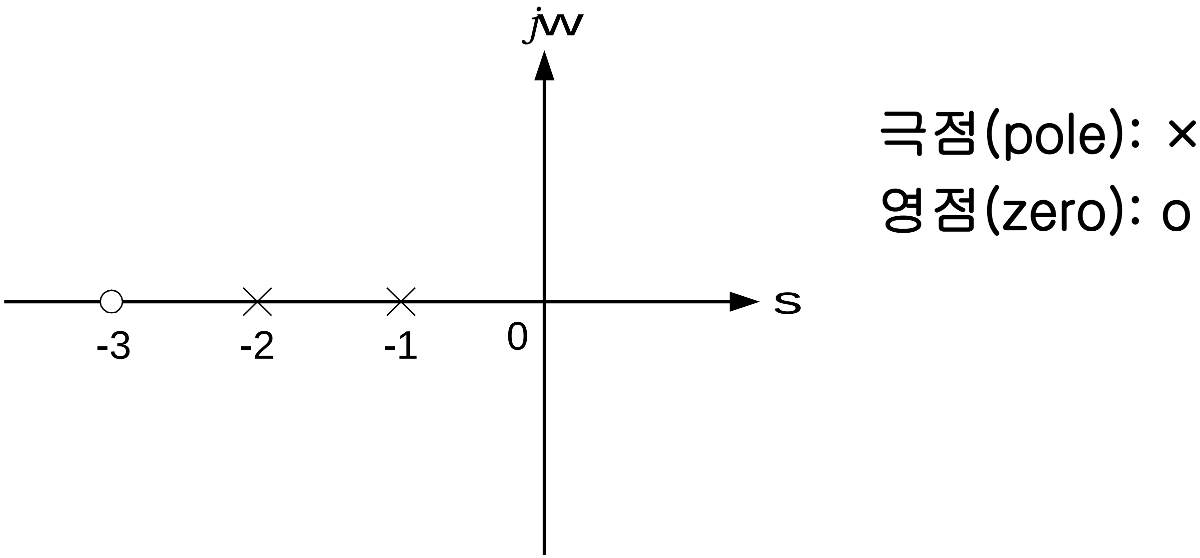


그림 2.5 시스템 (2.54)의 극점-영점 표시

[예제] 다음 그림과 같은 질량-스프링 기계 시스템이 초기조건, $x(0)=0,\ \dot{x}(0)=0$ 인 상태에서 단위임펄스 힘 δ(t)에 의해 작동할 때의 시스템 응답 $x(t)$ 를 구하고자 한다.
(단, 마찰력은 없다고 가정)

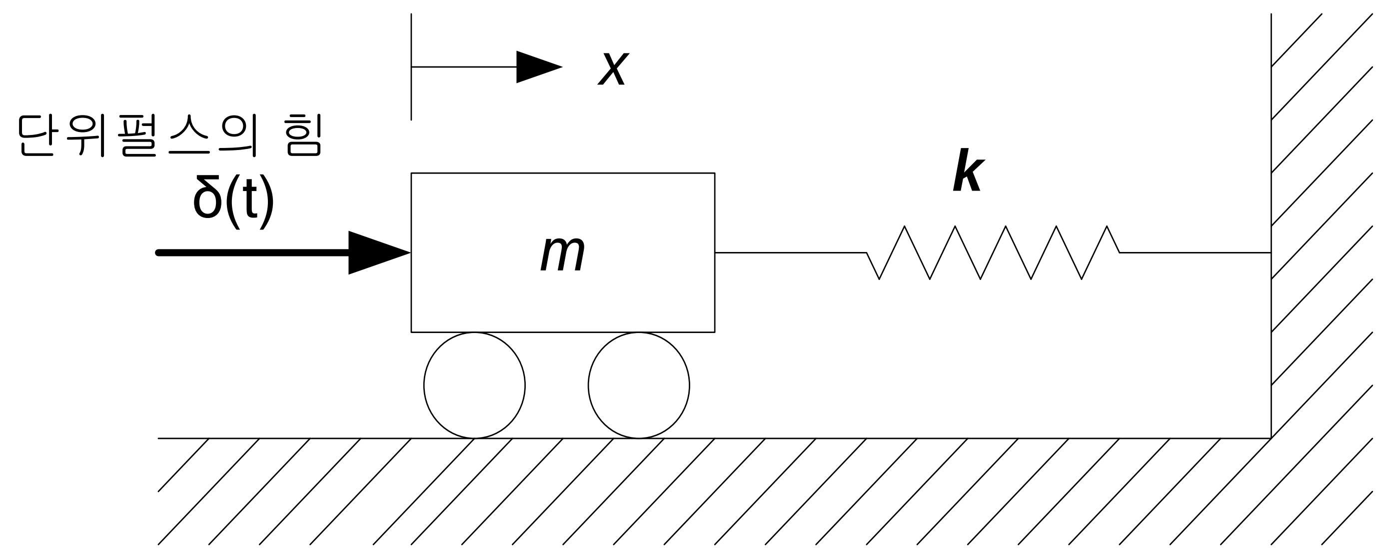


그림 2.3 질량-스프링 기계 시스템

Newton의 제 2 법칙 이용

$$m\ddot{x} + kx = \delta(t)$$

위 식을 Laplace 변환하면,

$$m\left\{s^2 X(s) - sx(0) - \dot{x}(0)\right\} + kX(s) = 1$$

초기조건 $x(0) = 0,\ \dot{x}(0) = 0$ 을 대입하고 X(s)에 대하여 식을 정리하면

$$X(s) = G(s) = \frac{1}{ms^2 + k}$$

다음 X(s)를 역 Laplace 변환하여 x(t)를 구한다.

$$x(t) = \frac{1}{\sqrt{mk}} \sin\sqrt{\frac{k}{m}}\, t$$

● 블록선도(block diagram)

실제 물리 시스템의 각 요소가 서로 어떻게 연관을 가지며 전 시스템의 성능에 어떻게 영향을 미치는가를 도식적으로 나타낸 선도.

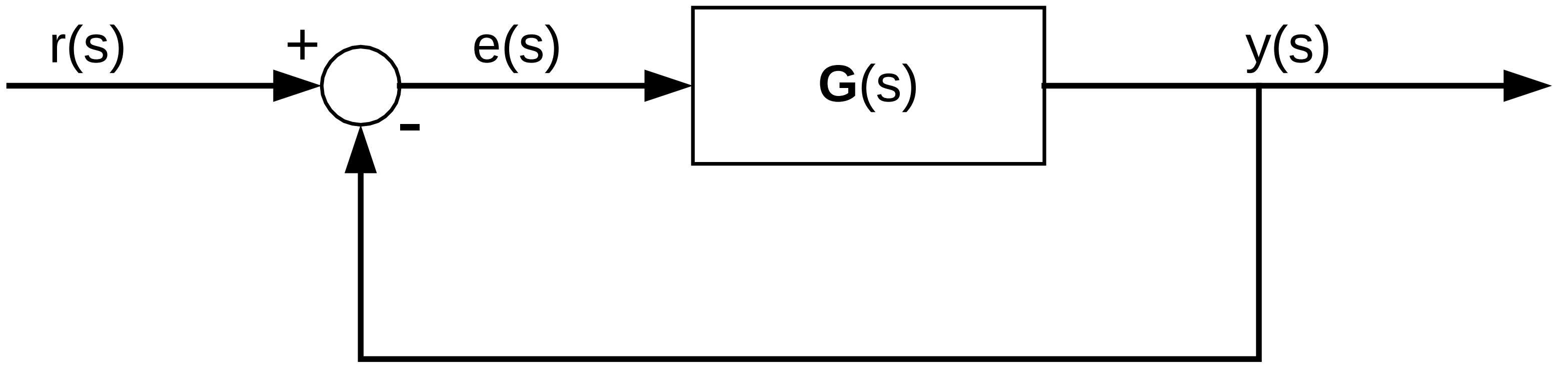


그림 2.6 폐루프 시스템의 블록선도

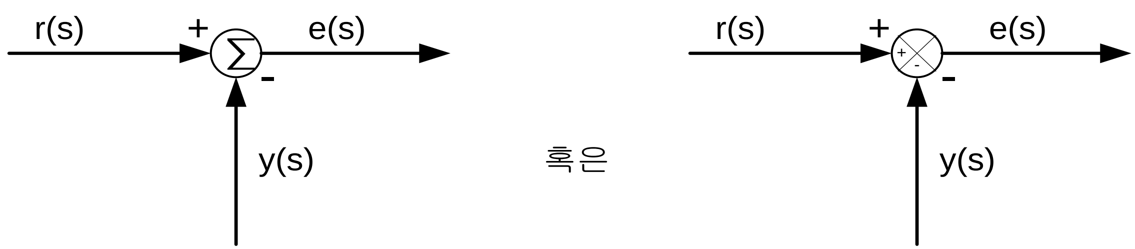


그림 2.7 블록선도에서 합산기호의 다른 표현방법

● 블록선도에 곱셈 연산이나 나눗셈 연산의 표시

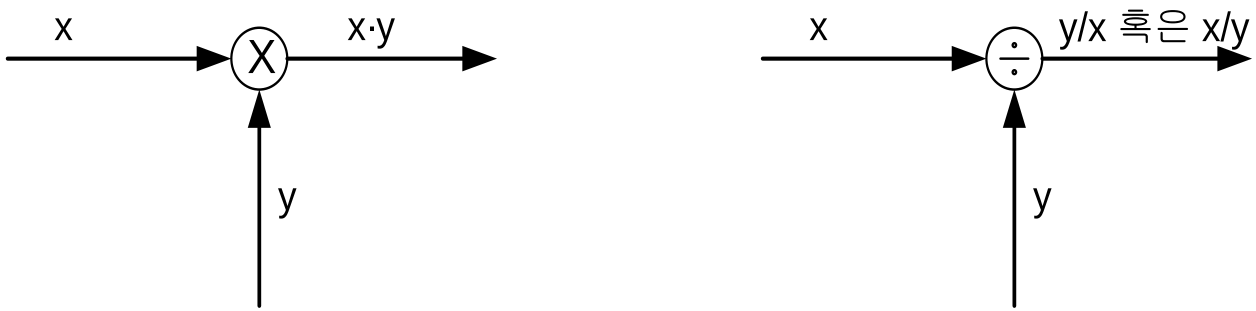


그림 2.8 블록선도에서 곱셈 및 나눗셈 연산의 표현방법

● 입력 및 출력의 개수가 2개 이상인 다변수 시스템

그림 2.9 다변수 시스템의 블록선도

● 피드백 제어시스템의 입출력 전달함수

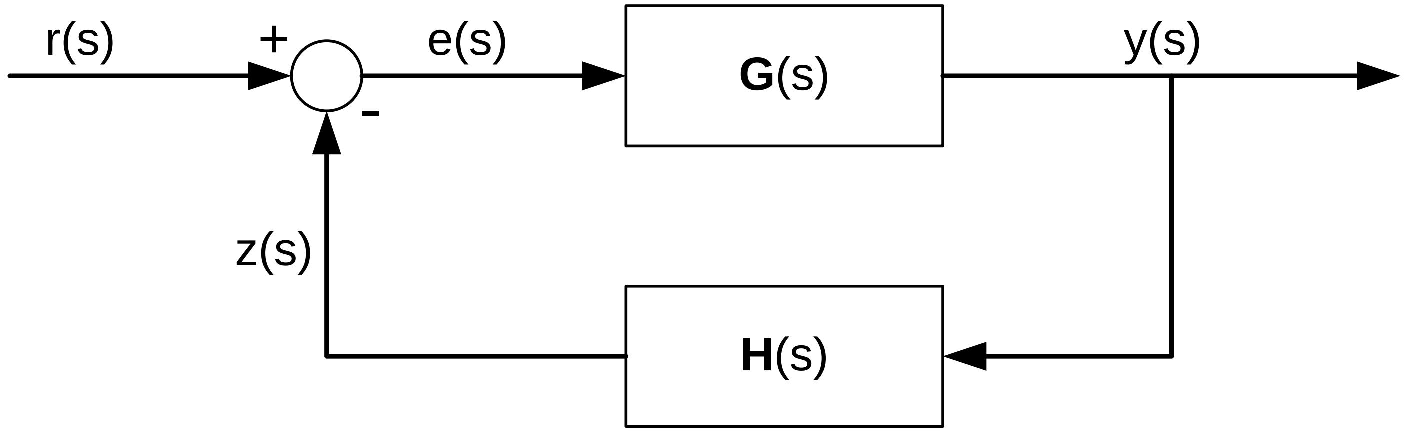


그림 2.11 피드백 제어시스템

제어입력 u(s) (=e(s))는

$$
\begin{aligned}
e(s) &= r(s) - z(s) \\
&= r(s) - H(s)\,y(s) \qquad (2.57)
\end{aligned}
$$

출력 y(s)와 제어입력 u(s) 사이의 관계식은

$$y(s) = G(s)e(s) \qquad (2.58)$$

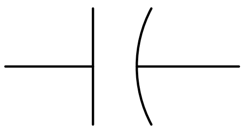
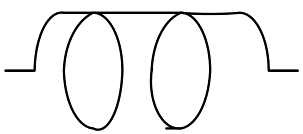
식 (2.57)을 식 (2.58)에 대입함으로써 전달함수 T(s)는

$$T(s) = \frac{y(s)}{r(s)} = \frac{G(s)}{1+G(s)H(s)} \qquad (2.59)$$

H(s) = 1 인 경우 : 단위 피드백 제어시스템

Electric Network Transfer function

Table 2.3 $$i = \frac{dq}{dt}$$

Component	Voltage-Current	Current-voltage	Voltage-charge	impedance $Z(s) = V(s)/I(s)$	Admittance Y(s)=I(s)/V(s)
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

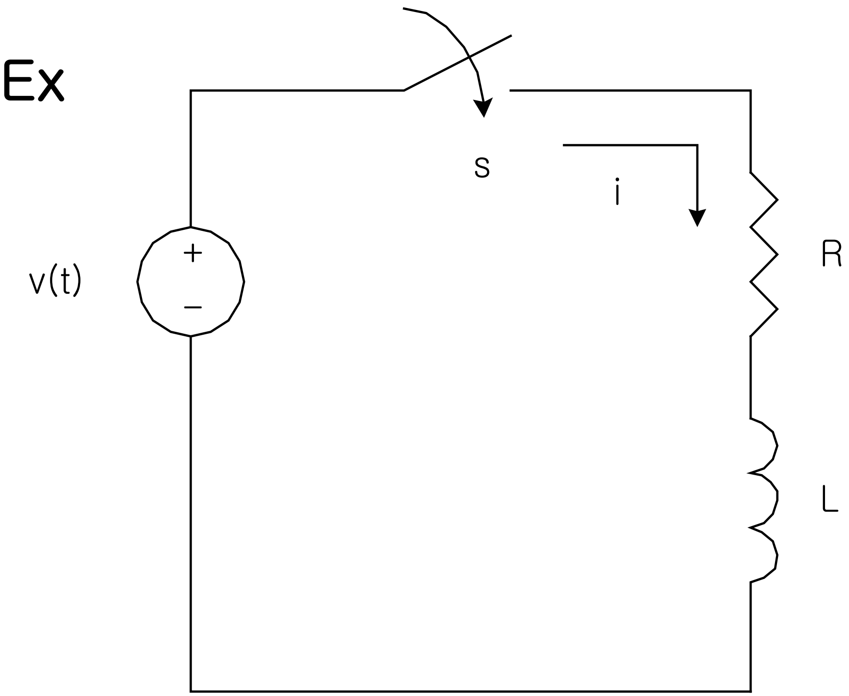
* Usage of Laplace Transform

$$F(s)=\int_{0}^{\infty} f(t)e^{-st}dt$$

Ex

By KVL

$$L\frac{di}{dt}+Ri=v(t)$$



$$< Method\,1 > By \;\; characteristic\; equation$$

$$L\frac{di}{dt} + Ri = 0 \;\; (Natural \;\; response) \qquad (1)$$

$$Let \;\; i = Ae^{st} \qquad (2)$$

$$\frac{di}{dt} = sAe^{st} \qquad (3)$$

$$(2) \;\; and \;\; (3) \quad \rightarrow \quad (1)$$

$$LsAe^{st} + RAe^{st} = (Ls + R)Ae^{st} = 0$$

$$For \;\; nontrival \;\; solution$$

$$Ls + R = 0, \qquad s = -\frac{R}{L} \qquad (4)$$

$$(4) \;\; \rightarrow \;\; (2) \qquad \therefore \qquad i = Ae^{-\frac{R}{L}t} \qquad (5)$$

$$If \;\; initial \;\; current \;\; (at \;\; t = 0^{+}) \; is \;\; I_0, \;\; then$$

$$i = A = I_0 \qquad (6)$$

$$(6) \;\; \rightarrow \;\; (5) \qquad \therefore \qquad i = I_0 e^{-\frac{R}{L}t}$$

$$< Method\,2 > \;\; By\;\; LT$$

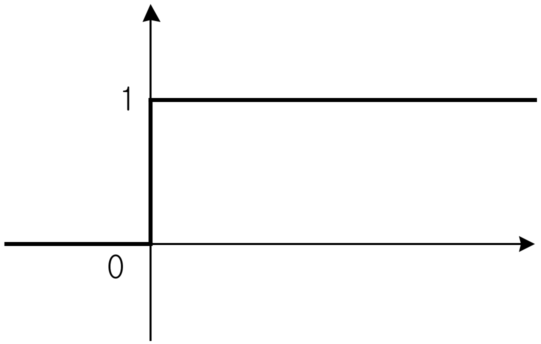
$$LsI(s) + RI(s) = V(s)$$

$$(Ls + R)I(s) = V(s)$$

$$\frac{I(s)}{V(s)} = T(s) = \frac{1}{Ls + R} \rightarrow Transfer\;\; function$$

$$If\;\; v(t) = u(t) : unit\, step\;\; fn.$$

$$V(s) = \frac{1}{s}$$



$$I(s) = T(s)V(s)$$

$$I(s) = \frac{1}{Ls + R} \cdot \frac{1}{s} = \frac{a}{Ls + R} + \frac{b}{s} = \frac{-L/R}{Ls + R} + \frac{1/R}{s}$$

$$\therefore i(t) = -\frac{1}{R} e^{-(R/L)t} + \frac{1}{R}$$

$$If\ \ L = 2H\ ,\ R = 10\Omega\ ,\ v(t) = 1V$$

$$I(s) = \frac{1}{2s+10}\cdot\frac{1}{s} = \frac{1/2}{s+5}\cdot\frac{1}{s}$$

By PFE

$$I(s) = \frac{1/2}{s+5}\cdot\frac{1}{s} = \frac{a}{s+5} + \frac{b}{s}$$

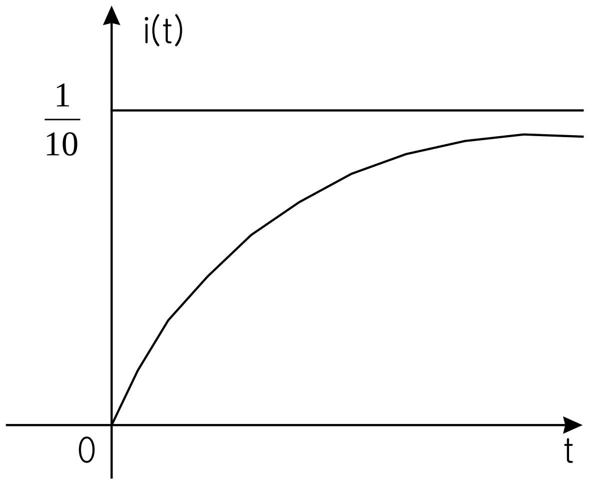
$$a = \left.\frac{1/2}{s}\right|_{s=-5} = -\frac{1}{10}$$

$$b = \left.\frac{1/2}{s+5}\right|_{s=0} = \frac{1}{10}$$

$$I(s) = -\frac{1}{10}\frac{1}{s+5} + \frac{1}{10}\frac{1}{s}$$

By ILT

$$i(t) = -\frac{1}{10}e^{-5t} + \frac{1}{10}\quad ,\quad t \geq 0$$



The amplification factor is dependent on both the source and load impedance

To be an ideal amplifier,

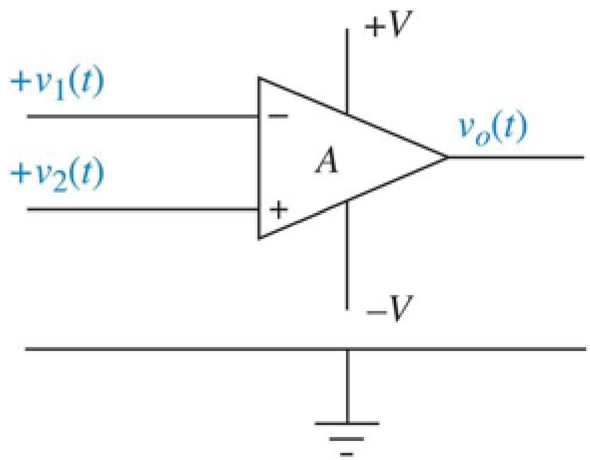
1) Differential Input $v_{in} = (v_2 - v_1)$

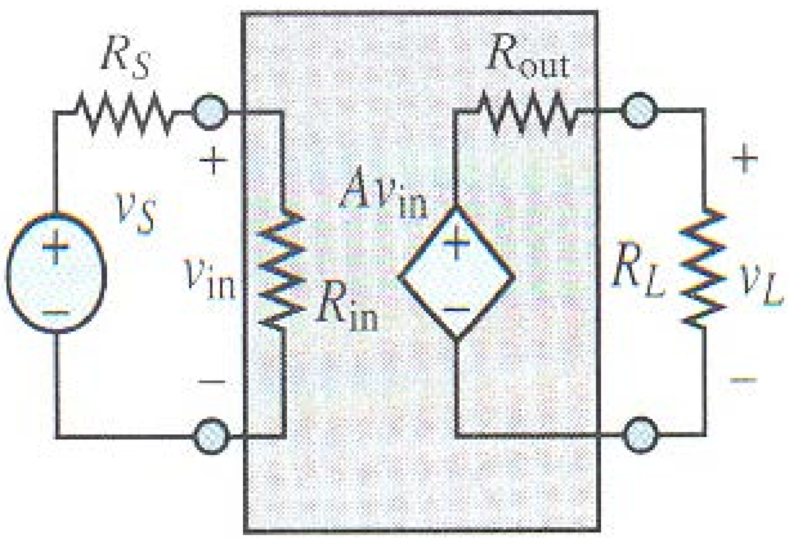
2) $R_{in} \rightarrow \infty$ and thus $v_{in} \approx v_s = v_2 - v_1$
3) $R_{out} \rightarrow 0$ and thus $v_L = Av_{in} = Av_s$.
4) $A = \infty$ (*ideal*)

$$v_o = A(v_2 - v_1)$$

$$\Leftarrow \quad v_{in}(t) = v_s \frac{R_{in}}{R_s + R_{in}}$$

$$v_L = Av_{in} \frac{R_L}{R_{out} + R_L}$$





The Operational Amplifier (Closed-Loop Model)

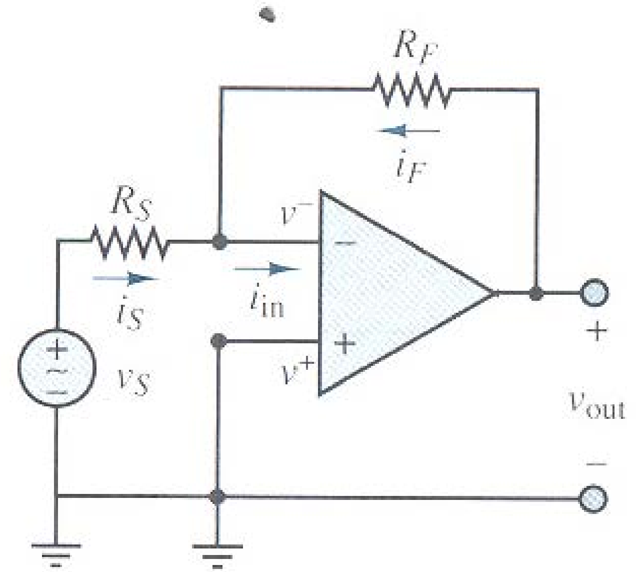
The Inverting Amplifier

$$i_S + i_F = i_{in}$$

$$i_S = \frac{v_S - v^-}{R_S}, \quad i_F = \frac{v_{out} - v^-}{R_F}, \quad i_{in} = 0$$

$$v_{out} = A_{V(OL)}\left(v^+ - v^-\right) = -A_{V(OL)} v^-$$

$$\text{or} \;\; v^- = -\frac{v_{out}}{A_{V(OL)}} (= 0 = v^+)$$



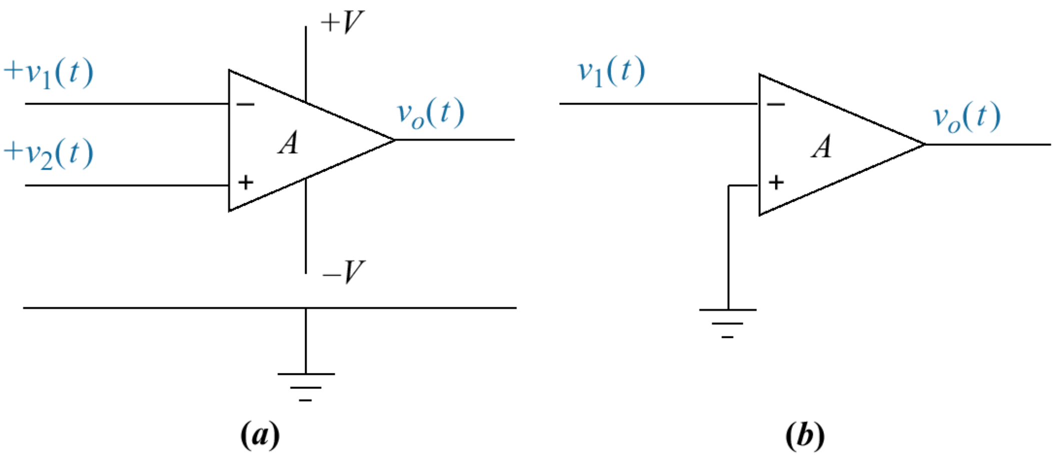
And, $i_S = -i_F$

$$\frac{v_S}{R_S} = -\frac{v_{out}}{R_F}$$

$$\therefore v_{out} = -\frac{R_F}{R_S} v_S$$

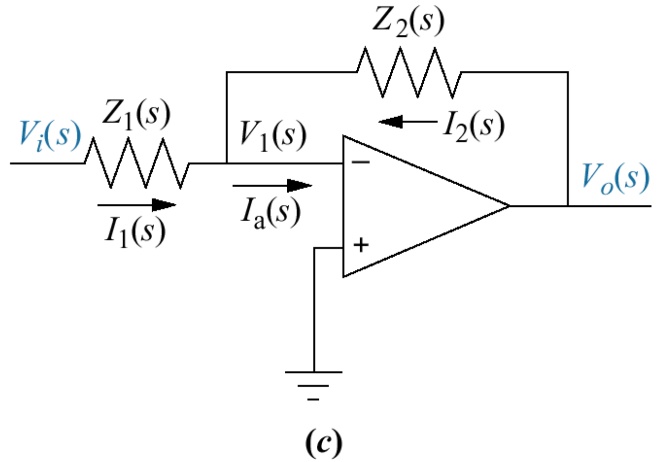
Figure 2.10

a. Operational amplifier;
b. schematic for an inverting operational amplifier;



$$I_1(s) = -I_2(s),\ I_1(s) = \frac{V_i(s)}{Z_1(s)},\ I_2(s) = \frac{V_o(s)}{Z_2(s)}$$

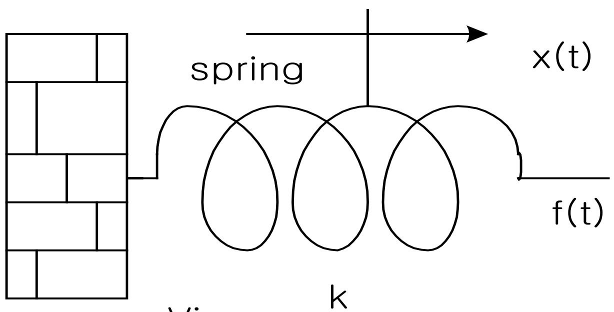
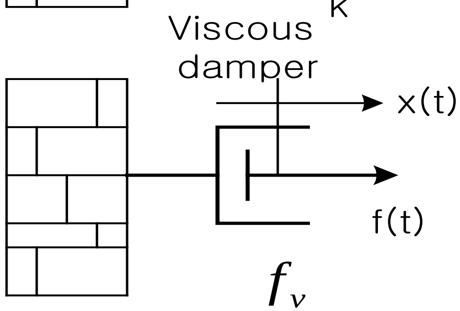
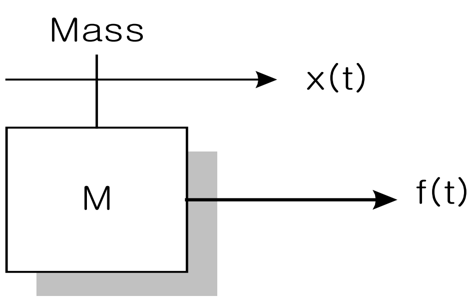
$$\therefore \frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$



c. Inverting operational amplifier configured for transfer function realization. Typically, the amplifier gain, A, is omitted.

Translational Mechanical system Transfer Functions

Table 2.4

Component	Force-velocity	Force-displacement	impedance $Z_M(s)=F(s)/X(s)$
	$f(t)=K\int_0^t v(\tau)\,d\tau$	$f(t)=Kx(t)$	K
	$f(t)=f_v v(t)$	$f(t)=f_v \frac{dx(t)}{dt}$	$f_v s$
	$f(t)=M\frac{dv(t)}{dt}$	$f(t)=M\frac{d^2x(t)}{dt^2}$	Ms^2

Examples 2.16 Transfer function-one equation of motion

Problem Find the transfer function X(s)/F(s), for the system Figure 2.15(a).

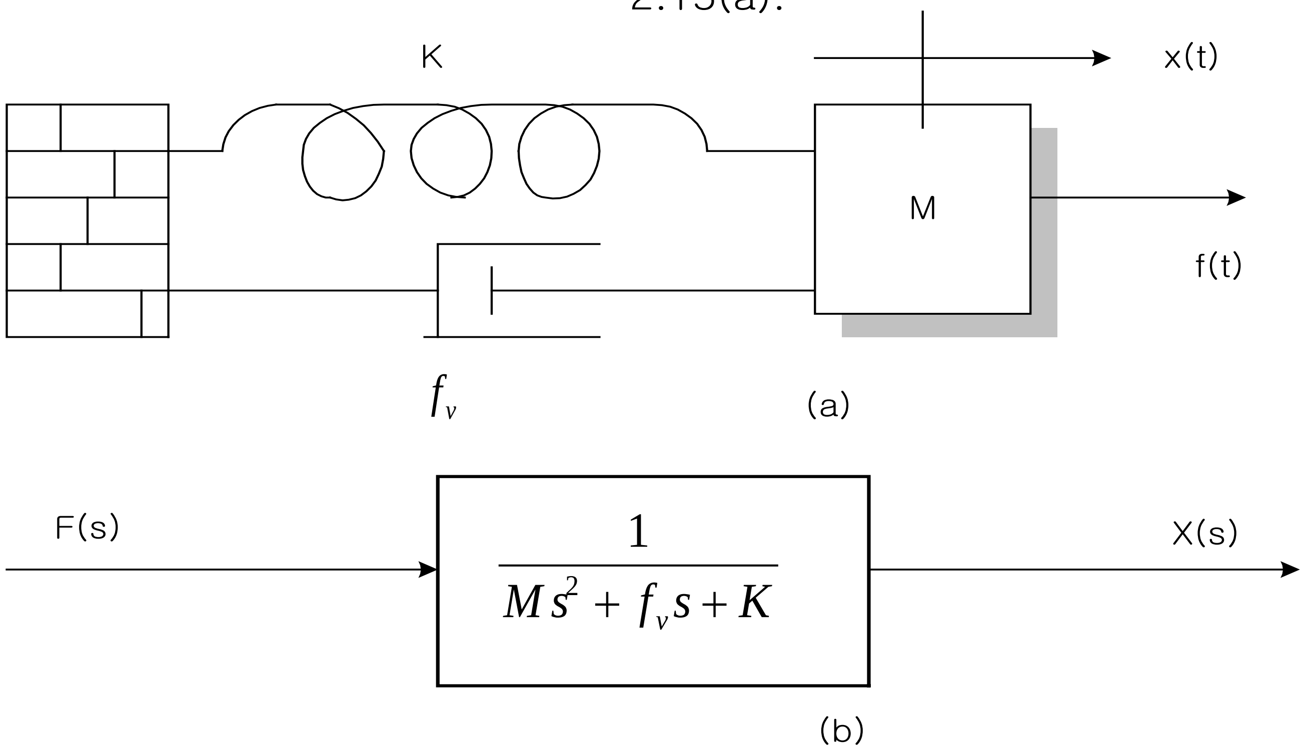


Figure 2.15
a. Mass spring, and damper system.
b. Block Diagram

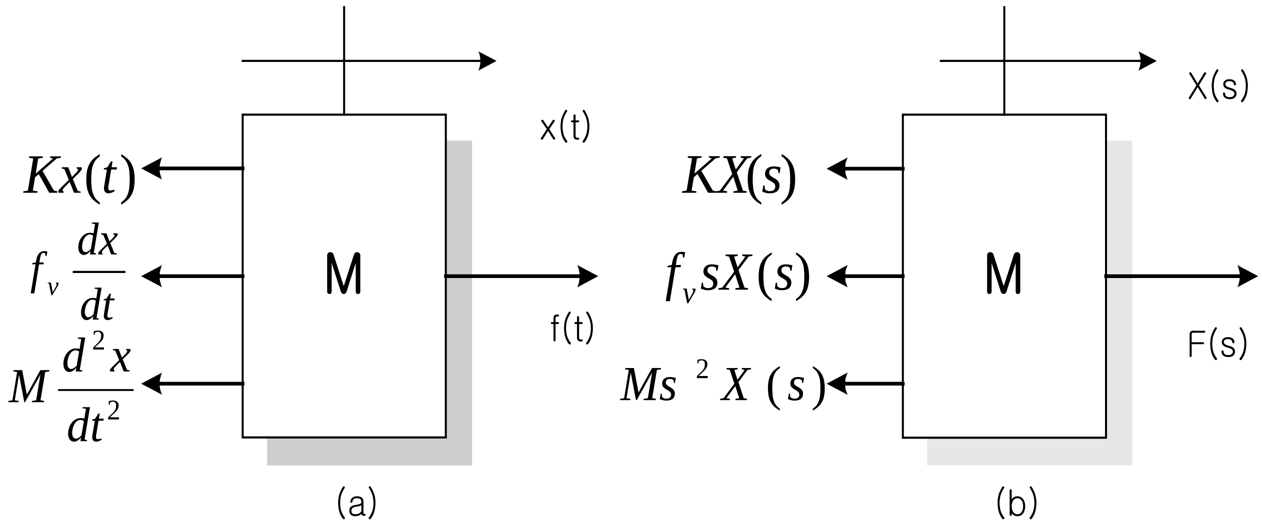


Figure 2.16
a. Free-body diagram od mass,spring, and damper system.
b. transformed free-body diagram

$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{dt} + kx(t) = f(t) \quad \text{---------} \quad (2.108)$$

$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s) \quad \text{---------} \quad (2.109)$$

$$(Ms^2 + f_v s + K)X(s) = F(s) \quad \text{---------} \quad (2.110)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K} \quad \text{---------} \quad (2.111)$$

Ex. 2.17 Transfer function – two degrees of freedom. Find $\dfrac{X_2(s)}{F(s)}$.

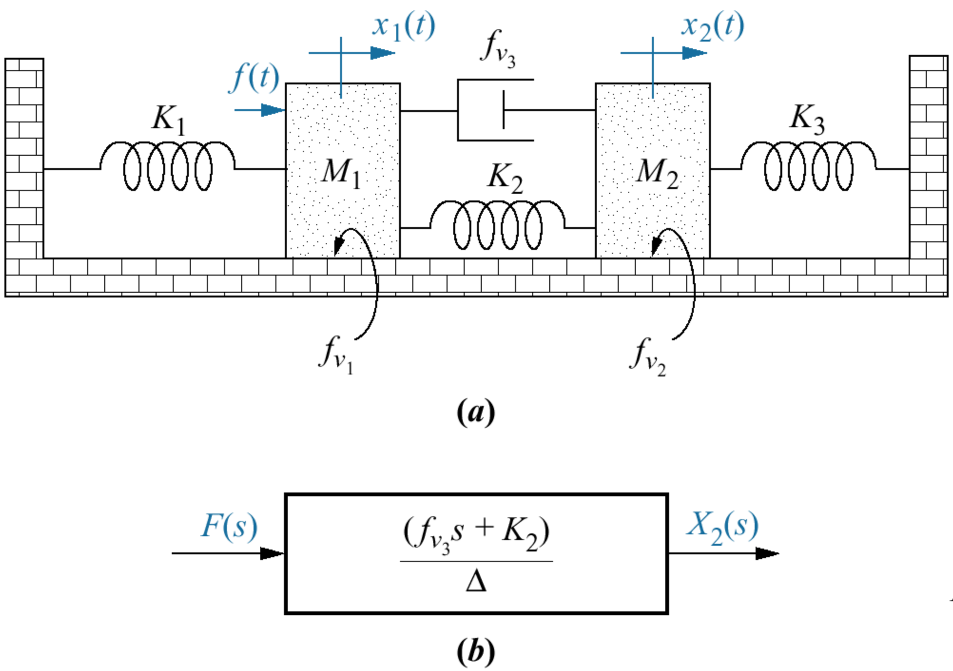
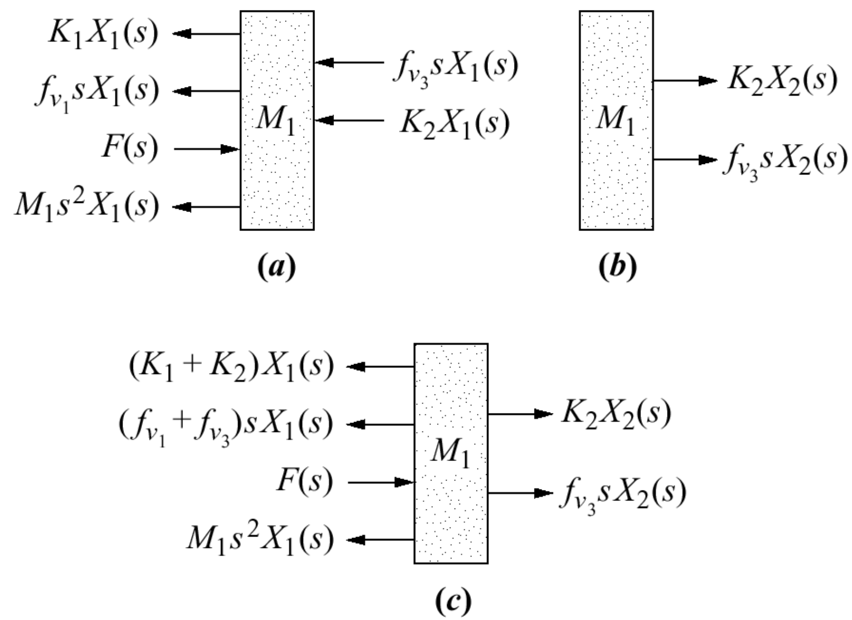


Figure 2.17
a. Two-degrees-of-freedom translational mechanical system
b. block diagram

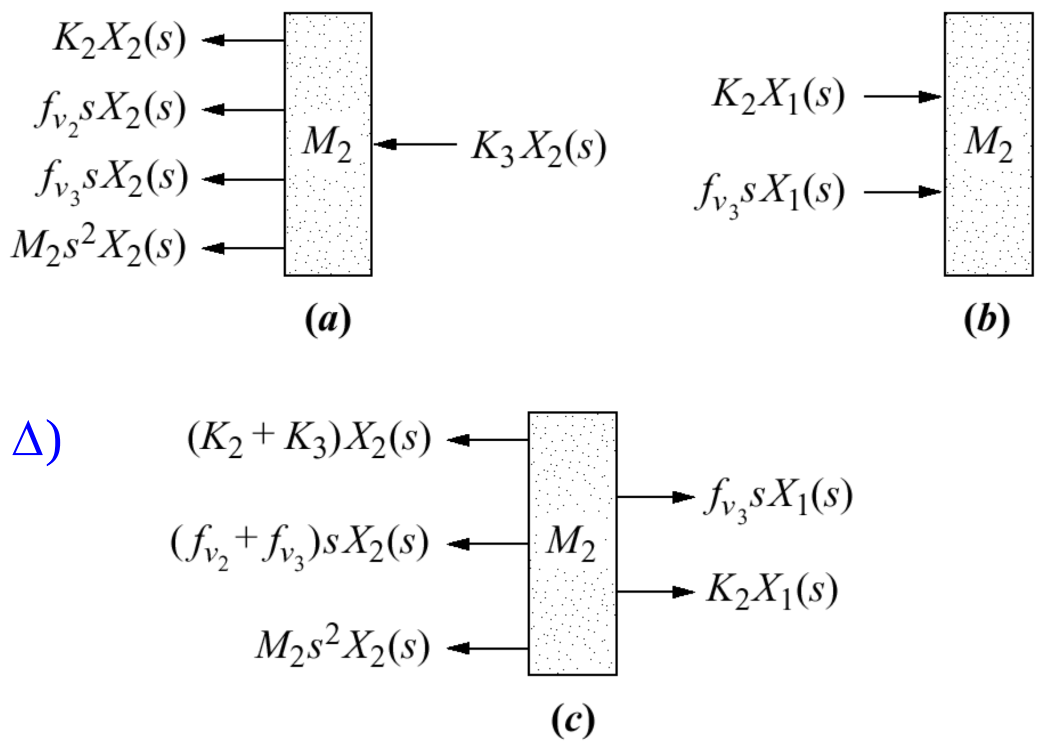
Sol)



$$\left[M_1 s^2 + (f_{v1} + f_{v3})s + (K_1 + K_2)\right] X_1(s) - (f_{v3}s + K_2) X_2(s) = F(s) \qquad (2.118a)$$

$$-(f_{v3}s + K_2) X_1(s) + \left[M_2 s^2 + (f_{v2} + f_{v3})s + (K_2 + K_3)\right] X_2(s) = 0 \qquad (2.118b)$$

$$\Delta = \left\| \begin{bmatrix} M_1 s^2 + (f_{v1} + f_{v3})s + (K_1 + K_2) & -(f_{v3}s + K_2) \\ -(f_{v3}s + K_2) & \left[M_2 s^2 + (f_{v2} + f_{v3})s + (K_2 + K_3)\right] \end{bmatrix} \right\|$$



$$A\mathbf{X} = \mathbf{F}$$

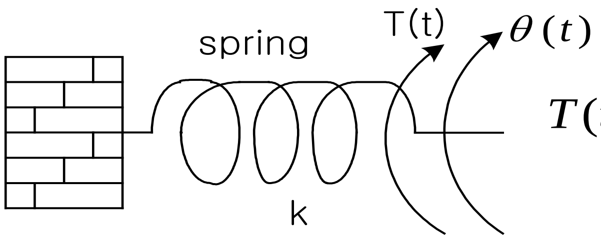
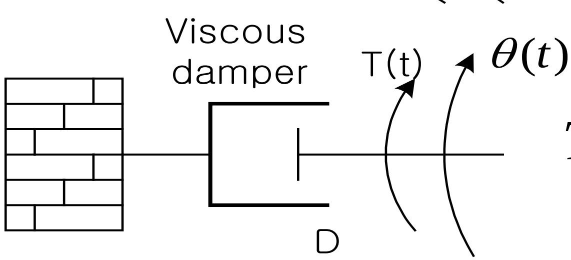
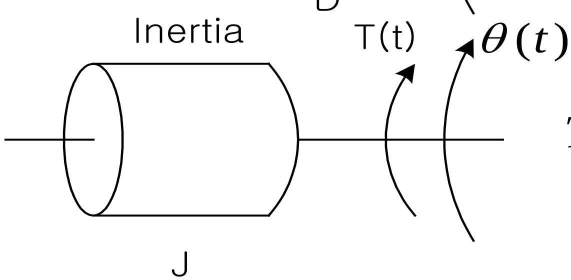
$$\mathbf{X} = A^{-1}\mathbf{F}$$

(단, $A^{-1} = [a_{ij}$의 여인자$]^{T}/\Delta$, $\det A = \Delta$)

$$\therefore \frac{X_2(s)}{F(s)} = G(s) = \frac{(f_{v3}s + K_2)}{\Delta}$$

Rotational Mechanical system Transfer Functions

Table 2.5

Component	Torque-angular velocity	Torque-angular displacement	impedance $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Ex 2. 19 Find the transfer function $\dfrac{\theta_2(s)}{T(s)}$

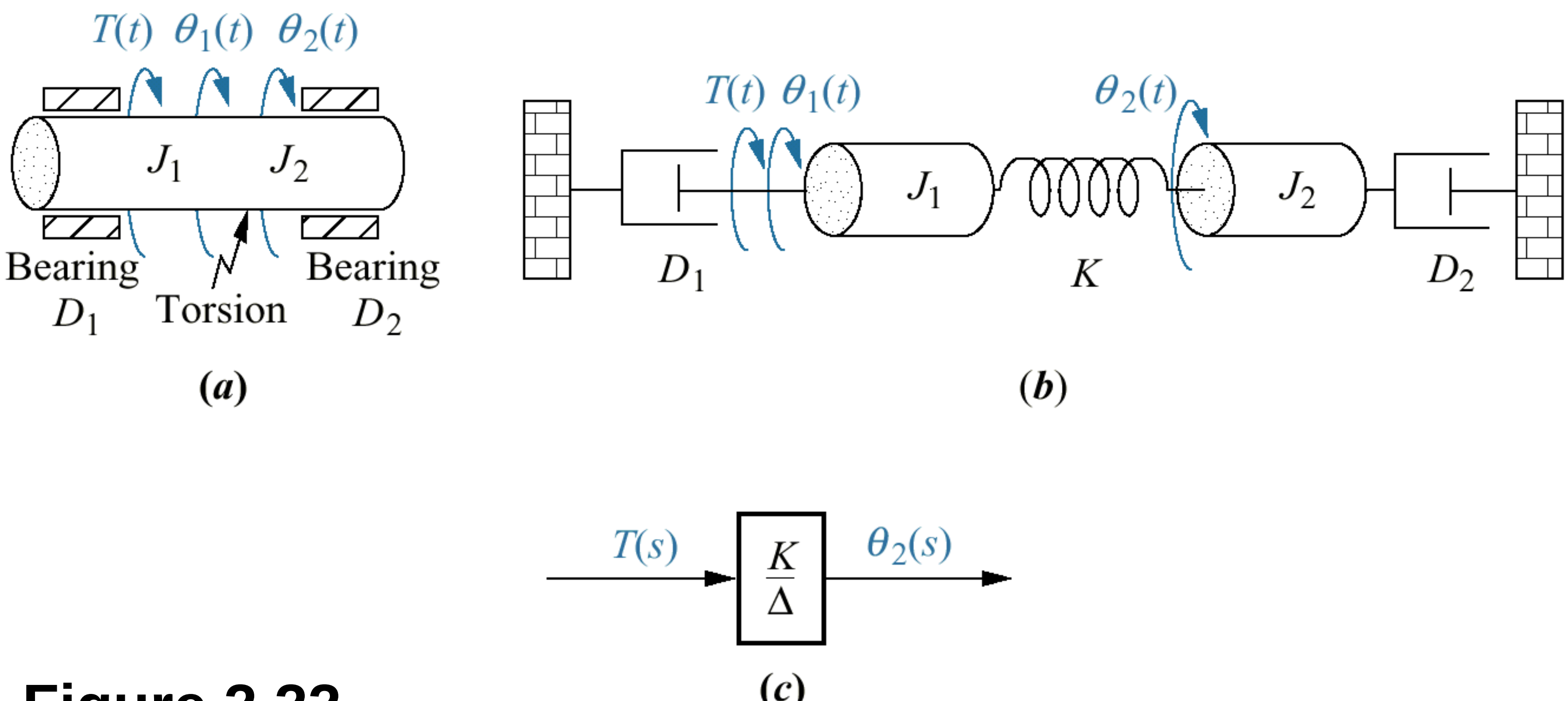


Figure 2.22
a. Physical system;
b. schematic; **c.** block diagram

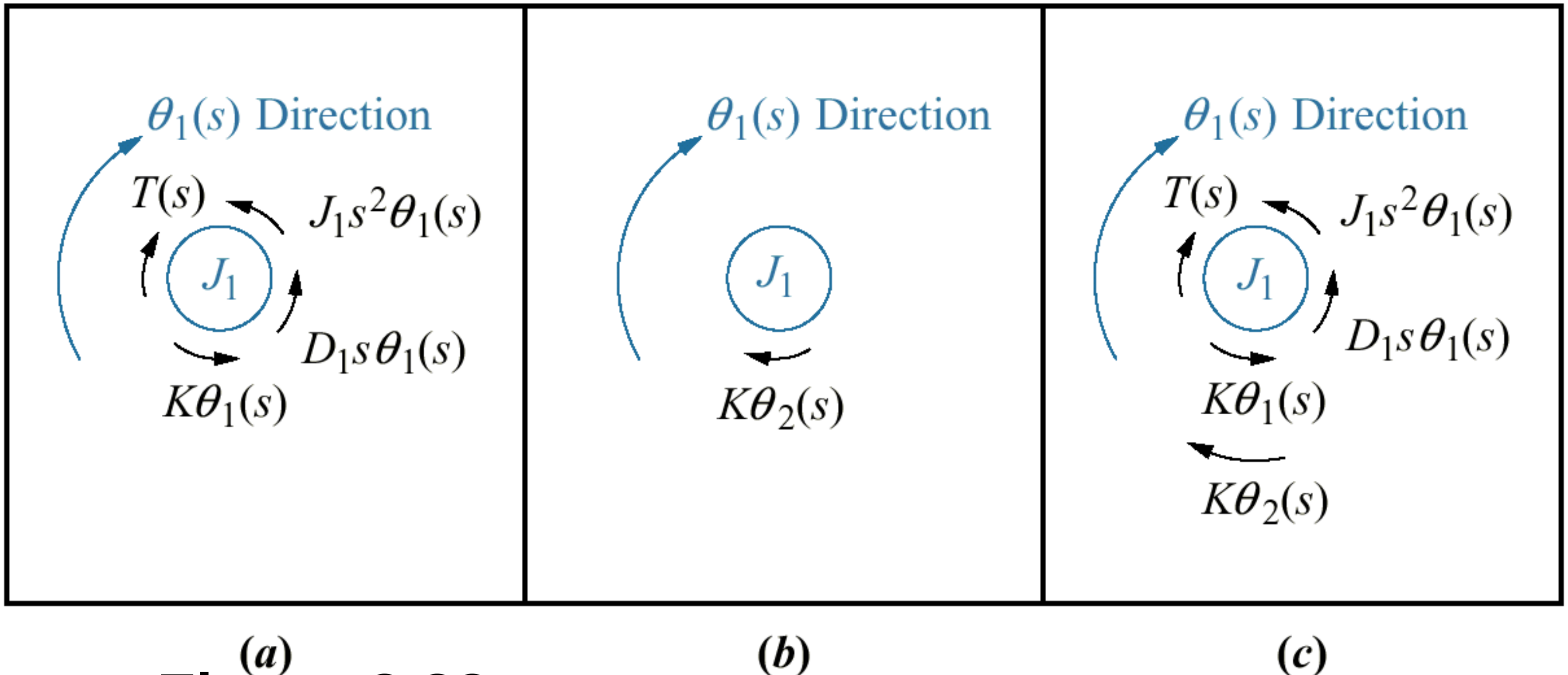


Figure 2.23

a. Torques on J_1 due only to the motion of J_1
b. torques on J_1 due only to the motion of J_2
c. final free-body diagram for J_1

$$\sum M = J\ddot{\theta}$$

$$T(t) - D_1\dot{\theta}_1 - K\theta_1 + K\theta_2 = J_1\ddot{\theta}_1$$

$$\Rightarrow T(s) - D_1 s\Theta_1(s) - K\Theta_1(s) + K\Theta_2(s) = J_1 s^2 \Theta_1(s)$$

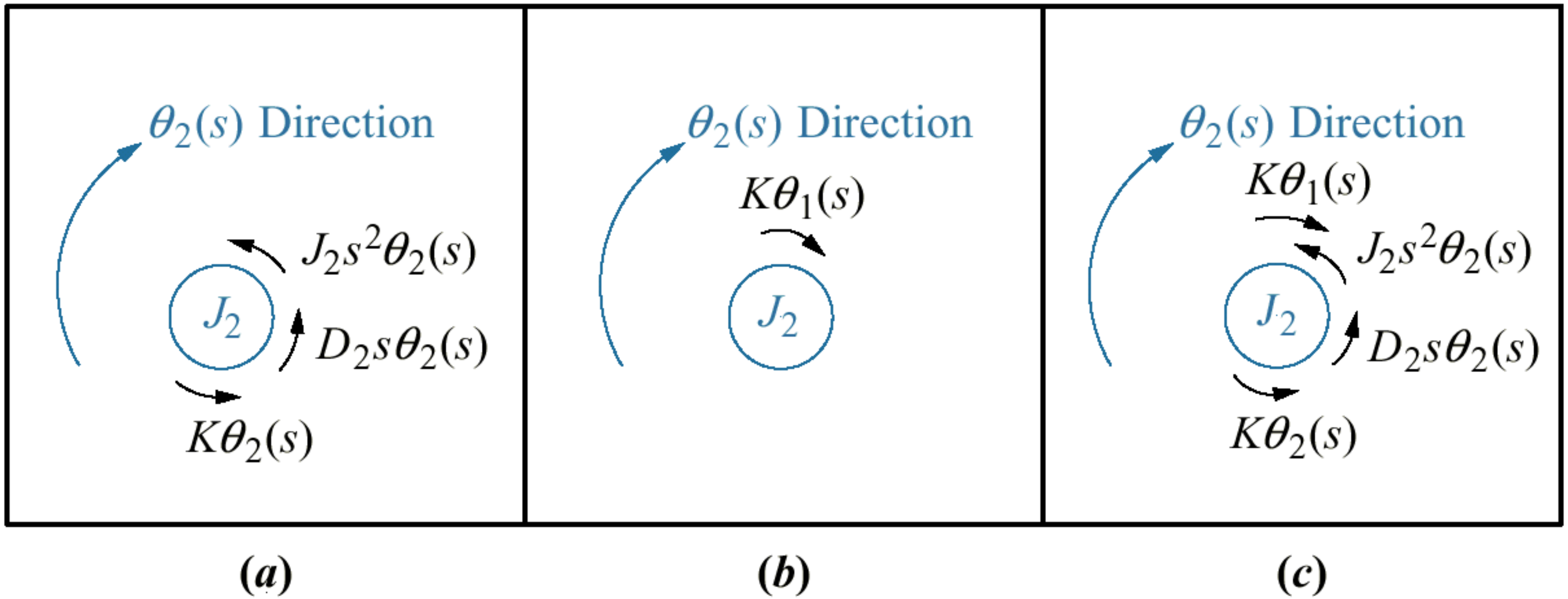


Figure 2.24

a. Torques on J_2 due only to the motion of J_2;
b. torques on J_2 due only to the motion of J_1
c. final free-body diagram for J_2

$$\sum M = J\ddot{\theta}$$

$$-D_2\dot{\theta}_2 + K\theta_1 - K\theta_2 = J_2\ddot{\theta}_2$$

$$\Rightarrow -D_2 s\Theta_1(s) + K\Theta_1(s) - K\Theta_2(s) = J_2 s^2\Theta_2(s)$$

$$(J_1 s^2 + D_1 s + K)\theta_1(s) \qquad - K\theta_2(s) = T(s) \tag{2.127a}$$

$$-K\theta_1(s) + (J_2 s^2 + D_2 s + K)\theta_2(s) = 0 \tag{2.127b}$$

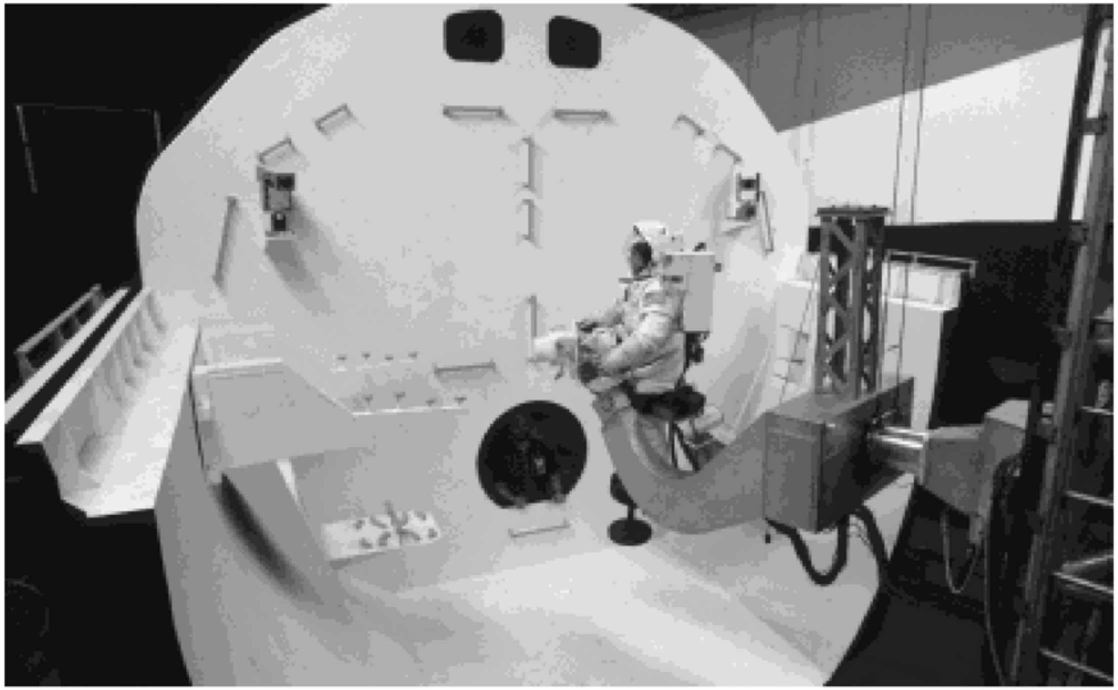
$$\frac{\theta_2(s)}{T(s)} = \frac{K}{\Delta} \tag{2.128}$$

$$\Delta = \begin{vmatrix} (J_1 s^2 + D_1 s + K) & -K \\ -K & (J_2 s^2 + D_2 s + K) \end{vmatrix}$$

$$\begin{bmatrix} \text{Sum of impedances} \\ \text{connected to the} \\ \text{motion at} \\ \theta_1 \end{bmatrix} \theta_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ \theta_1 \text{ and } \theta_2 \end{bmatrix} \theta_2(s) = \begin{bmatrix} \text{Sum of applied} \\ \text{torques at} \\ \theta_1 \end{bmatrix} \tag{2.129a}$$

$$-\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ \theta_1 \text{ and } \theta_2 \end{bmatrix} \theta_1(s) + \begin{bmatrix} \text{Sum of impedances} \\ \text{connected to the} \\ \text{motion at} \\ \theta_2 \end{bmatrix} \theta_2(s) = \begin{bmatrix} \text{Sum of applied} \\ \text{torques at} \\ \theta_2 \end{bmatrix} \tag{2.129b}$$

Figure 2.34
NASA flight
simulator
robot arm with
electromechanical
control system
components



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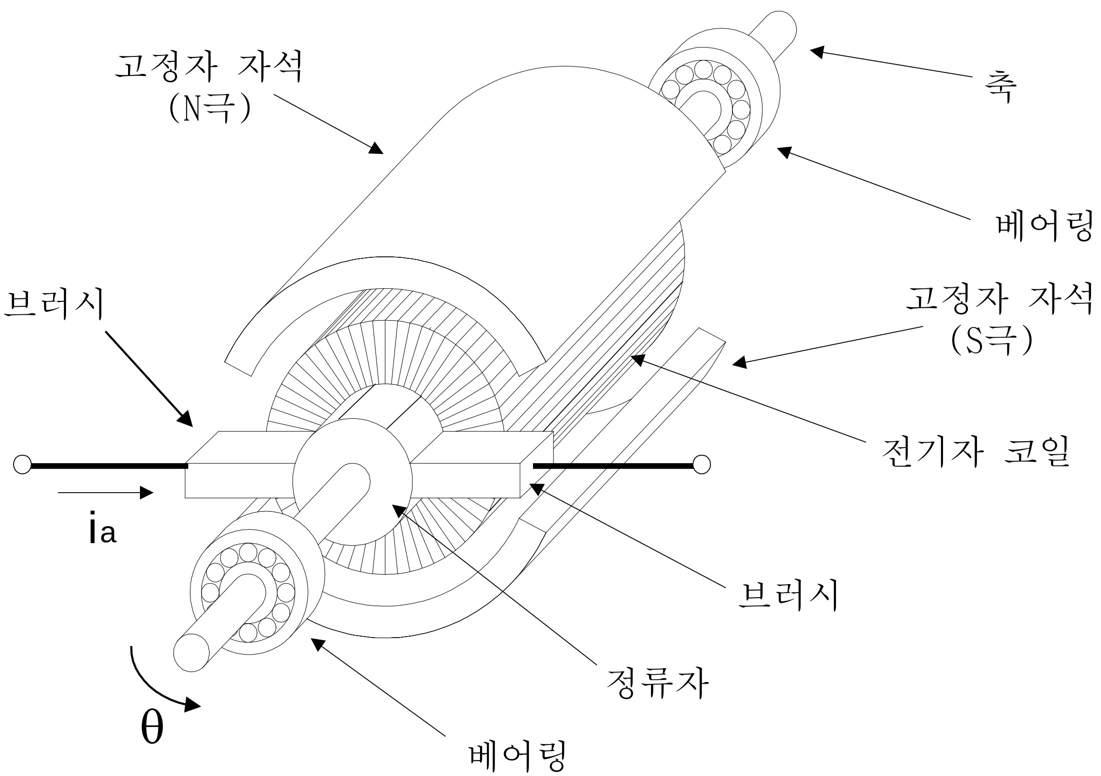
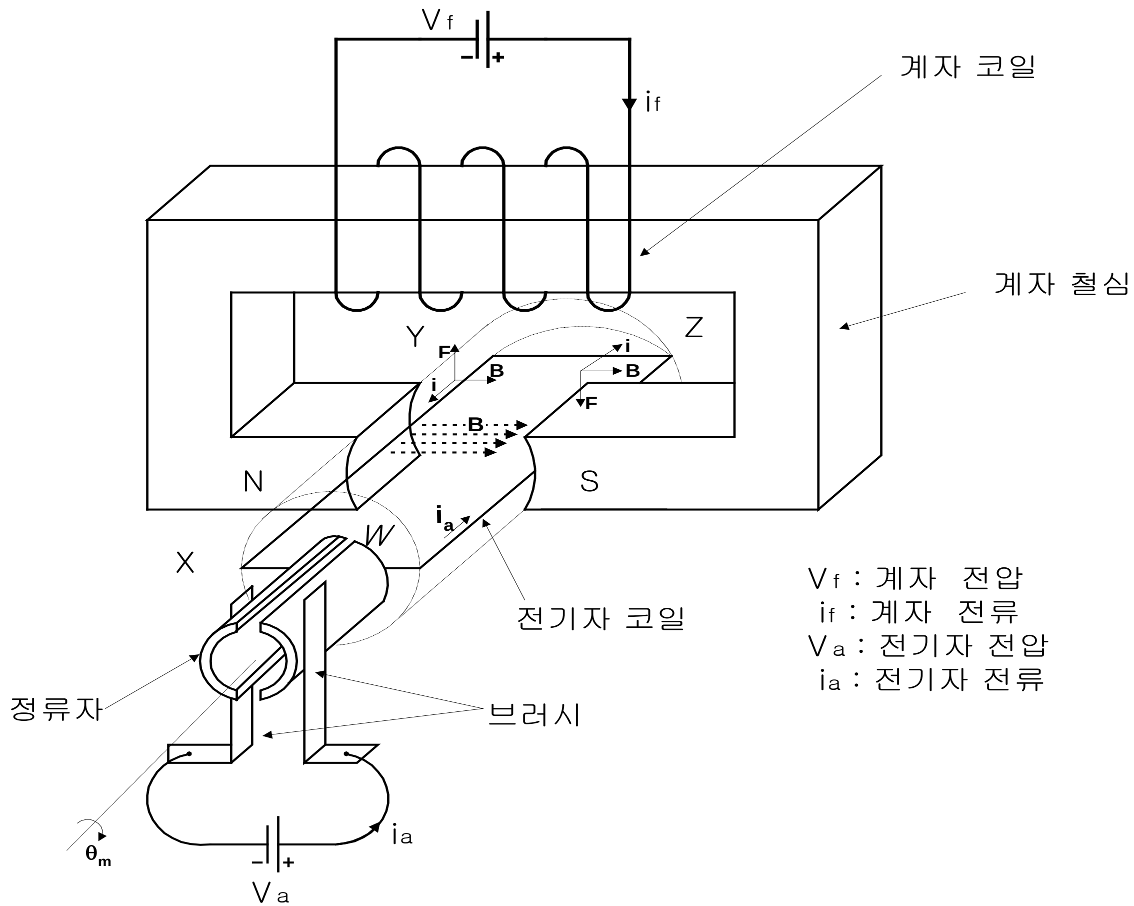
DC Servo Motor

● N극과 S극의 고정자 자석(stator magnet)이 정류자 (commutator) 및 전기자(armature) 철심 주변으로 둘러 싸여 있는 것



DC 서보 모터의 구조

● 계자(field) 철심을 이용하여 구성된 것



DC 서보 모터의 구동원리

● 모터에서 발생하는 토크의 크기

전류 i_a에 의한 전하 q의 속도를 v, 자장의 세기를 B라고 하면, 이 때 발생하는 Lorentz 힘 F는 다음과 같다.

$$F = qv \times B \tag{8.1}$$

$$dF = dq\frac{dx}{dt} \times B = i_a dx \times B \tag{8.2}$$

● 전기자 코일의 길이 l 까지에 의해 발생하는 힘 F

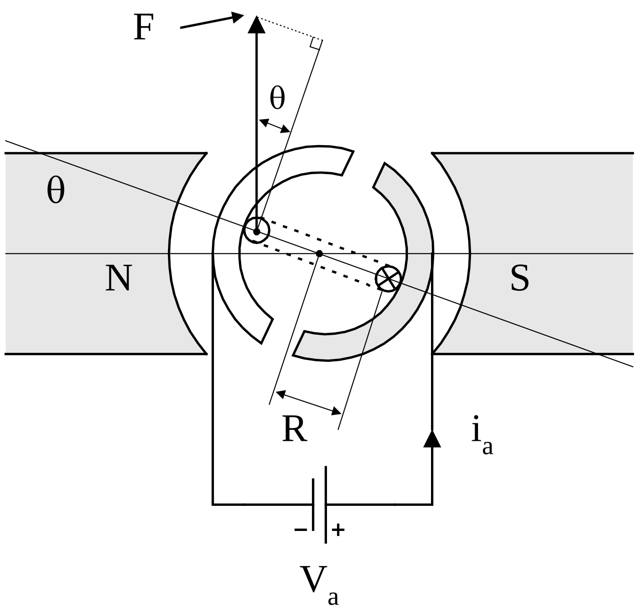
$$F = i_a(\int_0^l dx) \times B = i_a l \times B \tag{8.3}$$

$$|F| = Bli_a \tag{8.4}$$

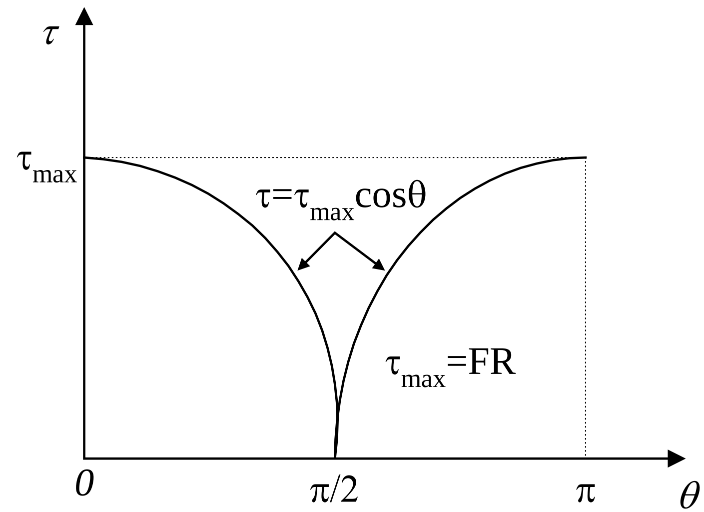
$$\tau = R|F| = RBli_a = K_t i_a \tag{8.5}$$

● 전기자 코일이 θ만큼 회전했을 때 발생되는 토크

$$\tau = F \cdot R\cos\theta = \tau_{\max}\cos\theta \qquad (8.6)$$



(a) 전기자 코일의 회전위치

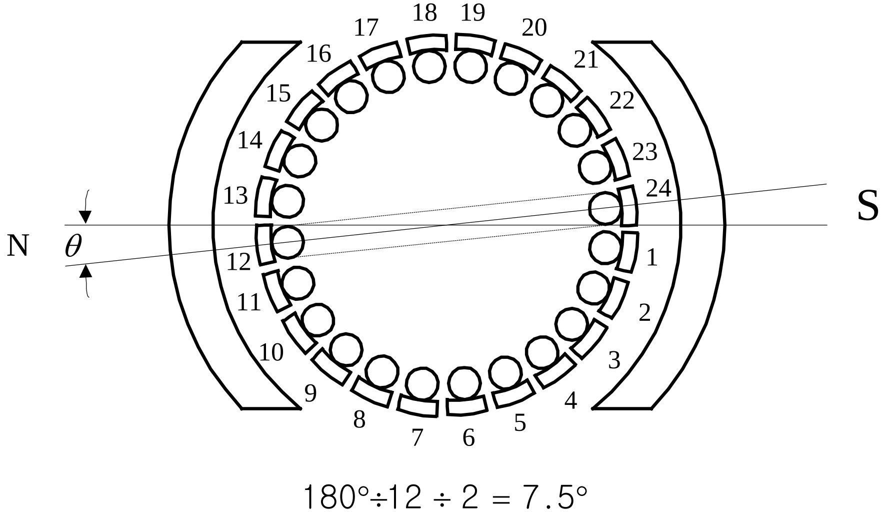


($\theta = \pi/2$ 에서 코일 전류 방향이 역전)

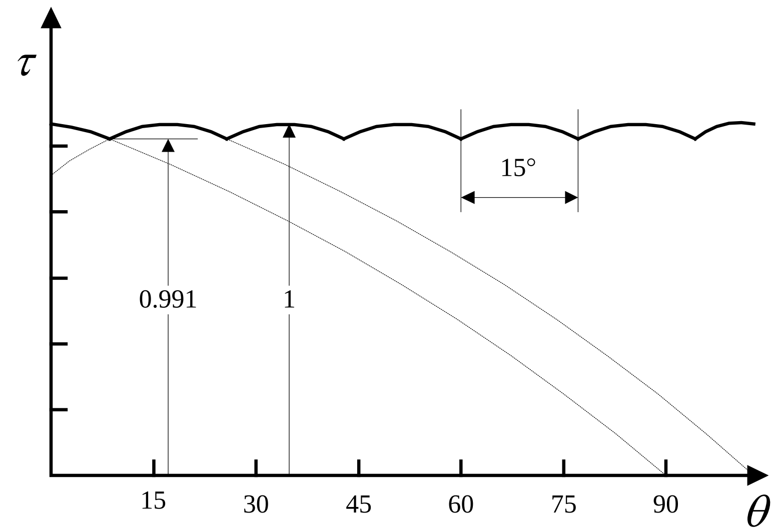
(b) 토크의 변동

전기자 코일의 회전 위치에 따른 토크의 변동

● 12개의 코일을 등간격으로 배치한 예



(a) 12개 전기자 코일의 간격



(b) 발생 토크

12개의 등간격 전기자 코일과 발생 토크

$$T_m = RBli_a = K_t i_a$$

$$v_b = BlR\frac{d\theta_m}{dt} = K_e\frac{d\theta_m}{dt}$$

$K_t = RBl$: 토크 상수[kgf. cm/A]

$K_e(= BlR)$: 역기전력(back electromotive force) 상수[V/krpm]

=> 단위계가 같으면 $K_t = K_e (= K)$

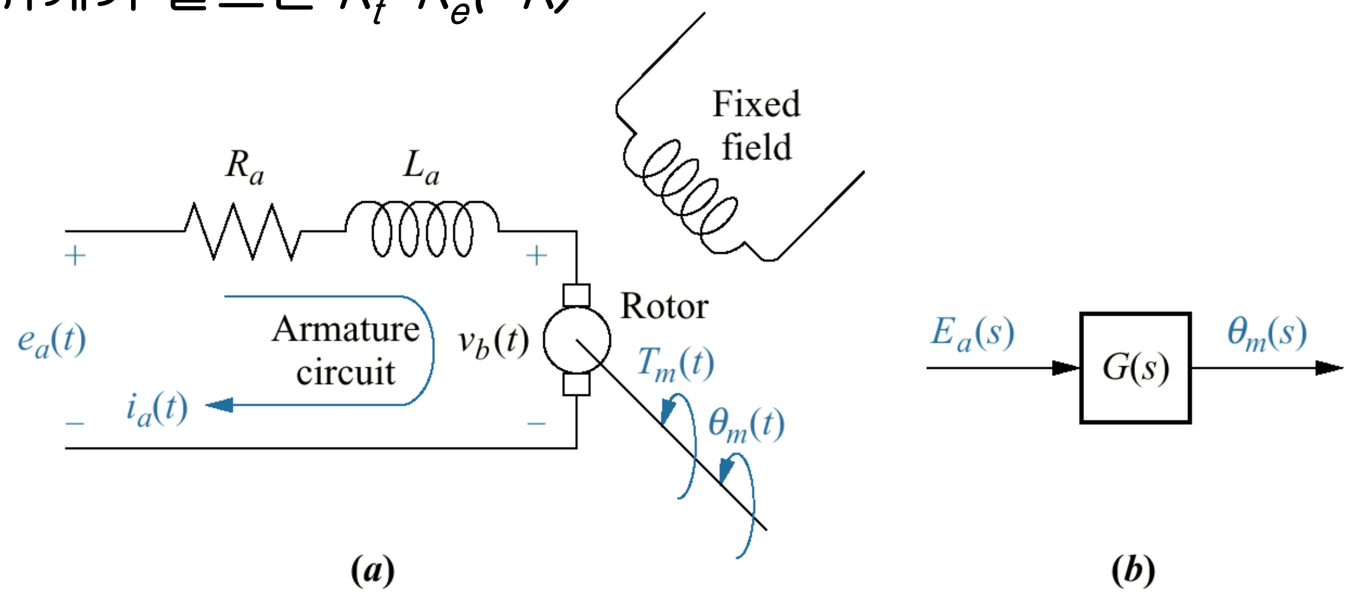
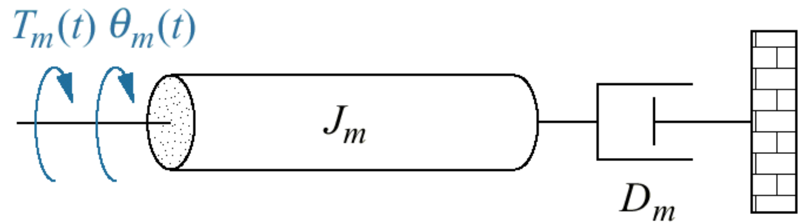


Figure 2.35 DC motor:
a. schematic[12]; **b.** block diagram

● Kirchhoff의 전압법칙을 적용한 전기회로의 동적 방정식

$$L_a \frac{di_a}{dt} + R_a i_a + v_b = e_a, \quad L_a \approx 0 \text{ 로 가정하면}$$

$$R_a i_a + K_e \dot{\theta}_m = e_a \qquad (1)$$



● DC 서보 모터의 동특성

$$T_m = T_l + T_d = K_t i_a, \qquad T_d \approx 0 \text{ 로 가정}$$

$$\sum M = J\ddot{\theta} \quad \Rightarrow \quad T_m - D_m \dot{\theta}_m = J_m \ddot{\theta}_m$$

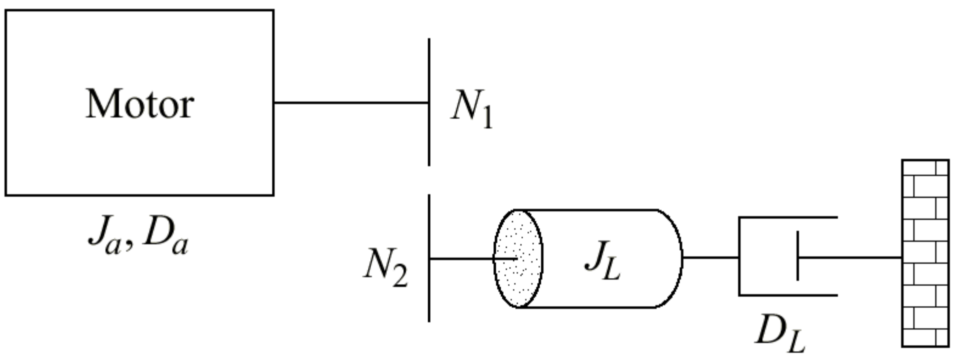
$$\therefore T_m = J_m \ddot{\theta}_m + D_m \dot{\theta}_m = K_t i_a = K_t \left(\frac{e_a - K_e \dot{\theta}_m}{R_a} \right) \qquad (2)$$

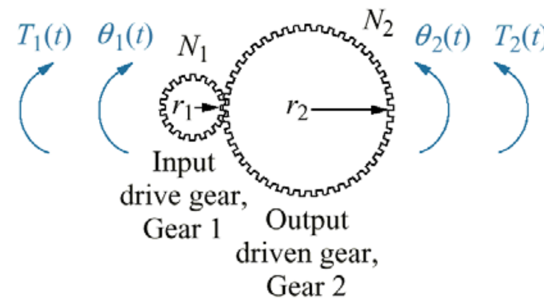
$$\text{식 (1),(2)로부터} \quad \Rightarrow \quad J_m \ddot{\theta}_m + \left(D_m + \frac{K_t K_e}{R_a} \right) \dot{\theta}_m = \frac{K_t}{R_a} e_a$$

$$\therefore G(s) = \frac{\theta_m(s)}{E_a(s)} = \frac{K_t / R_a}{J_m s^2 + (D_m + K_t K_e / R_a)s} = \frac{K_t /(R_a J_m)}{s[s + (D_m + K_t K_e / R_a) / J_m]}$$

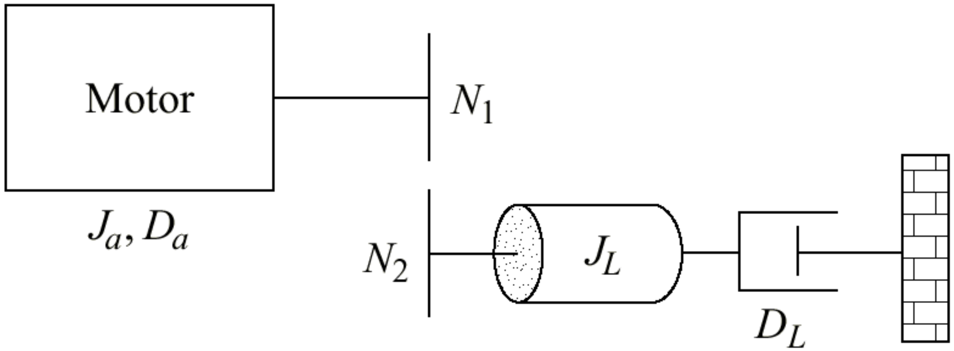
$$= \frac{K}{s(s+\alpha)}$$

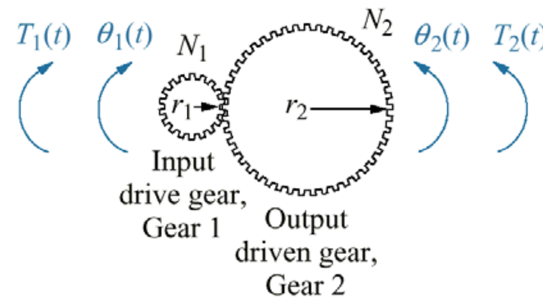
- *DC motor driving a rotational mechanical load*
  Transfer function Gears with loss





$$J_{eff} = J_a + J_L \left(\frac{N_1}{N_2} \right)^2 ; \quad D_{eff} = D_a + D_L \left(\frac{N_1}{N_2} \right)^2$$





$$r_1\theta_1 = r_2\theta_2, \quad N_1\theta_1 = N_2\theta_2, \quad \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = n < 1$$

- *Energy by input torque T_1 = Energy by delivered torque T_2*

$$T_1\theta_1 = T_2\theta_2 \quad \therefore \frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = n$$

$$T = T_m + T_1 = T_m + nT_2 = \left(J_a + n^2 J_L\right)\ddot{\theta}_1 + \left(D_a + n^2 D_L\right)\dot{\theta}_1 = J_{eff}\ddot{\theta}_1 + D_{eff}\dot{\theta}_1$$

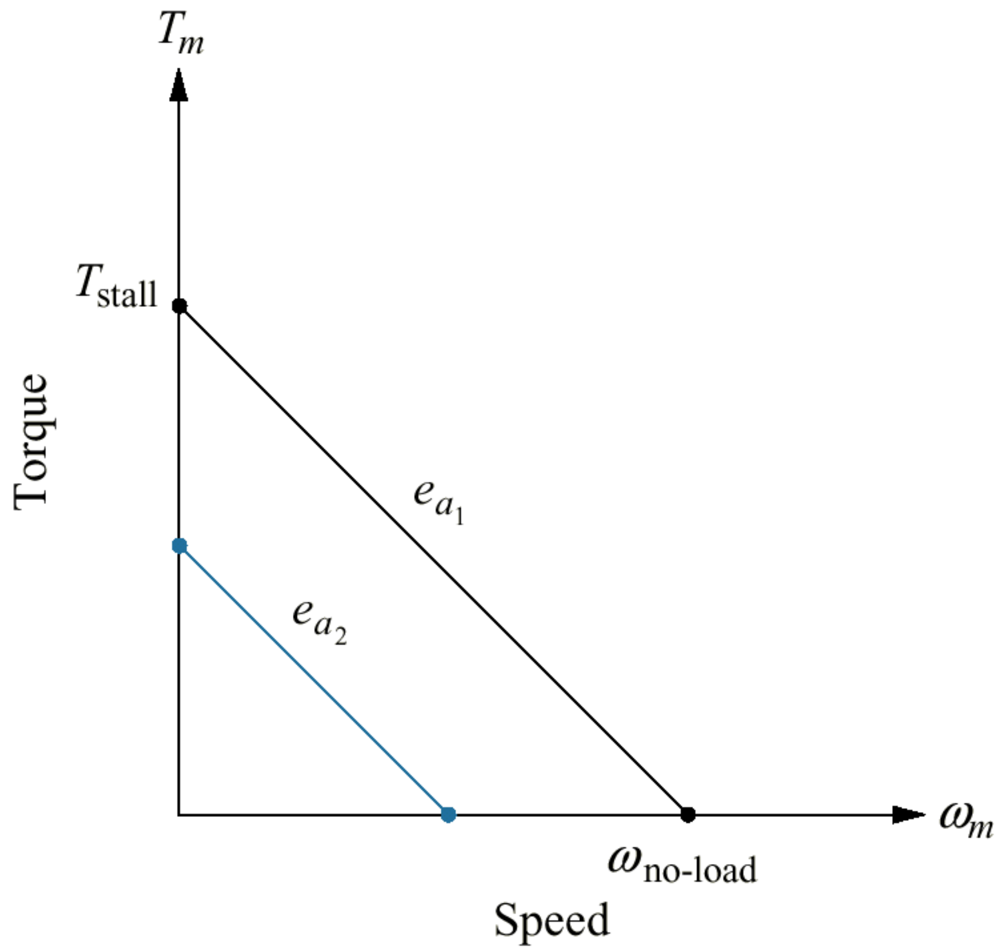
$$\because T_m = J_a\ddot{\theta}_1 + D_a\dot{\theta}_1, \quad T_2 = J_L\ddot{\theta}_2 + D_L\dot{\theta}_2 = n(J_L\ddot{\theta}_1 + D_L\dot{\theta}_1)$$

$$J_{eff} = J_a + J_L\left(\frac{N_1}{N_2}\right)^2; \quad D_{eff} = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

Figure 2.38
Torque-speed curves with an armature voltage, e_a, as a parameter

$$T_m = K_t i_a = K_t\left(\frac{E_a - K_e\dot{\theta}_m}{R_a}\right)$$

$$T_m = -\frac{K_t K_e \dot{\theta}_m}{R_a} + \frac{K_t E_a}{R_a}$$



Home Work #2-2 (Due date: two weeks from today)

1. Solve Problem 22(21) on page 97(100) in the text book 7th ed *(6th edition)

2. Solve Problems 24(23) on page 98(100) in the text book.

3. Solve Problem 27(26) on page 98(101) in the text book.

4. Solve Problem 43(40) on page 101(103) in the text book.

5. Solve Problem 45(42) on page 101(103) in the text book.

6. Solve Problem 59(62) on page 103(108) in the text book 7th ed.