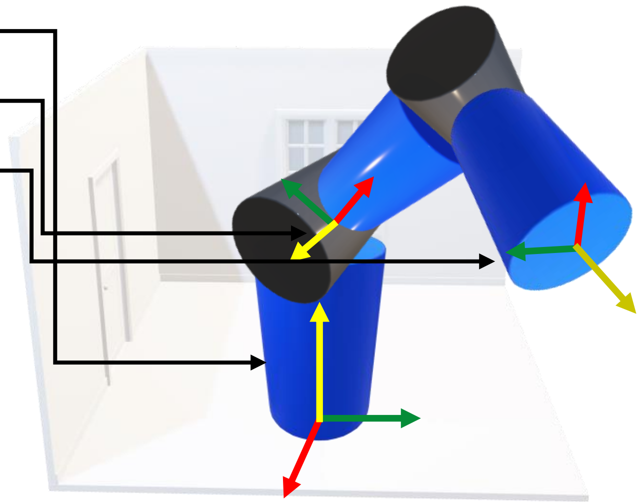
Kinematics and transfer matrix

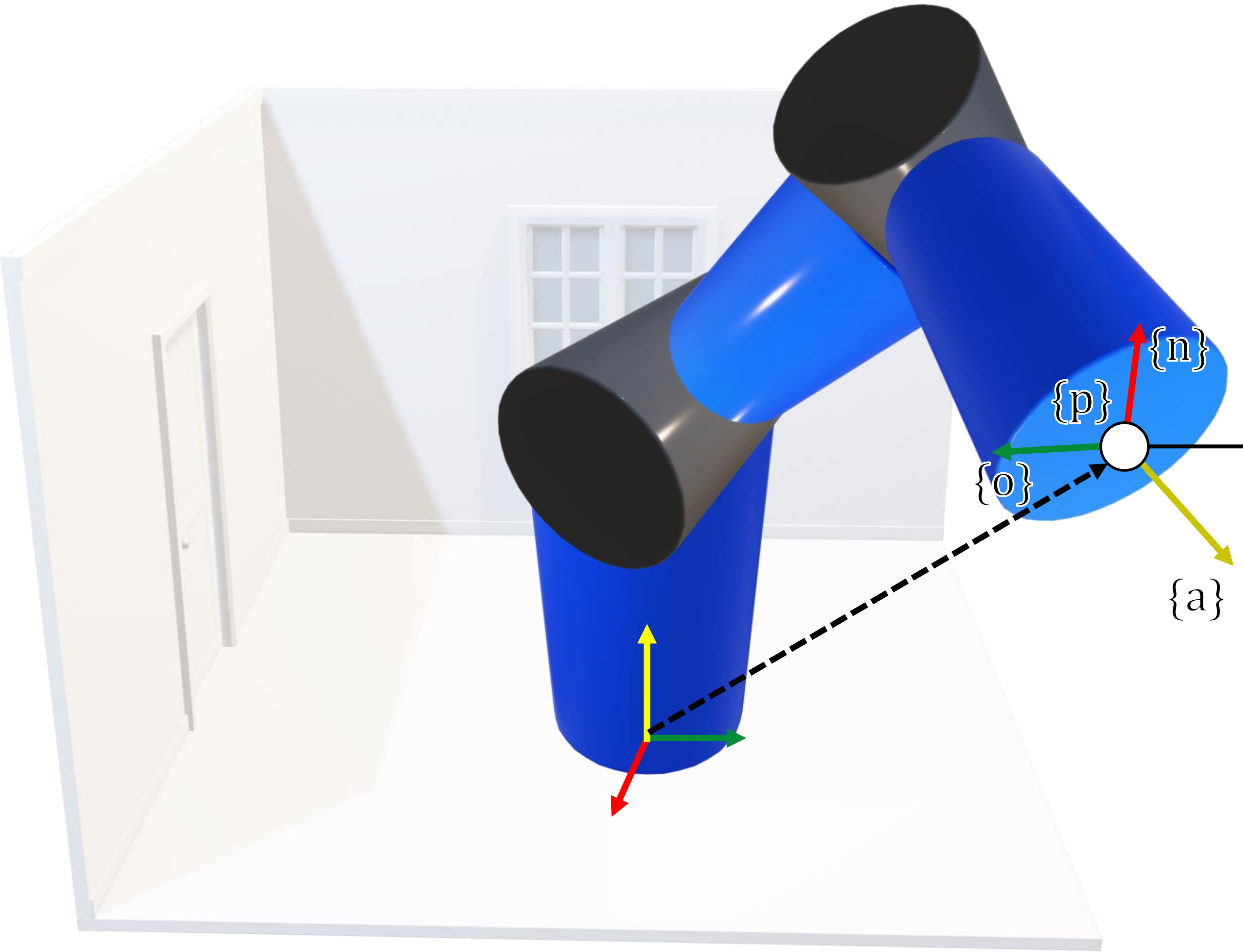
#1

The cartesian space

1. Cartesian space?
   - The coordinate system which specified by perpendicular numerical coordinates
     - ✓ Mostly, the 3-dimensional space where we belong
   - In Robotics, coordinates are classified into three types
     - ✓ Base coordinates
     - ✓ Axis coordinates
     - ✓ Tool coordinates



The cartesian space - 4 X 4 Matrix



{n, o, a, p} (also known as {x, y, z, p})

({n}, {o}, {a}, {p})∈ R^{3X1}

$$T_E = \begin{bmatrix} R & P \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \{n\} & \{o\} & \{a\} & \{p\} \\ & \mathbf{0}_{1\times 3} & & \mathbf{1} \end{bmatrix}_{4\times 4}$$

R: Rotation matrix, contains x/y/z unit vector of the current point.

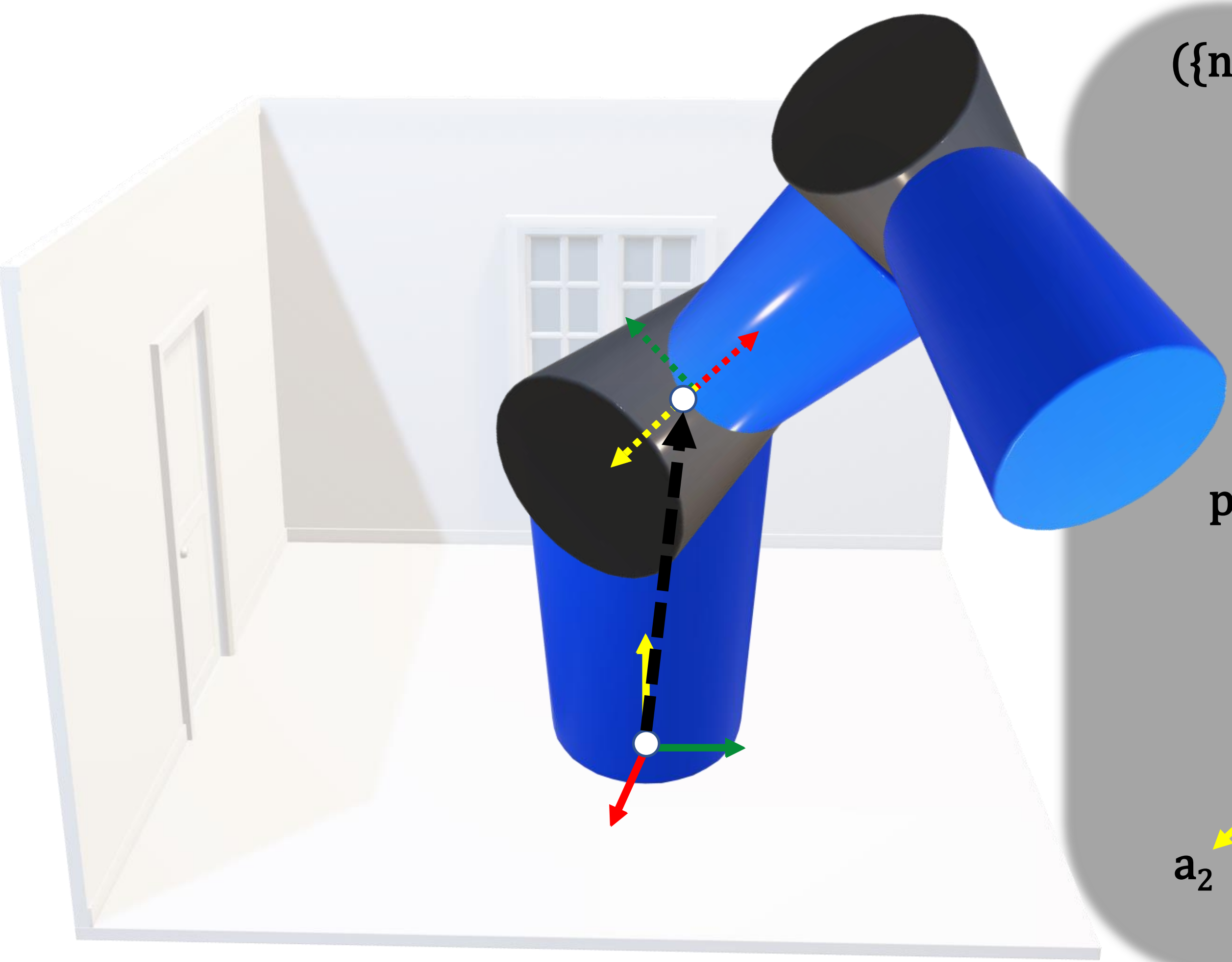
Called SO(3), 'special orthogonal group'

$R^T R = I$

$|R| = \mathbf{1}$

P: Position array, contains xyz position array vector from the certain point to the current point.

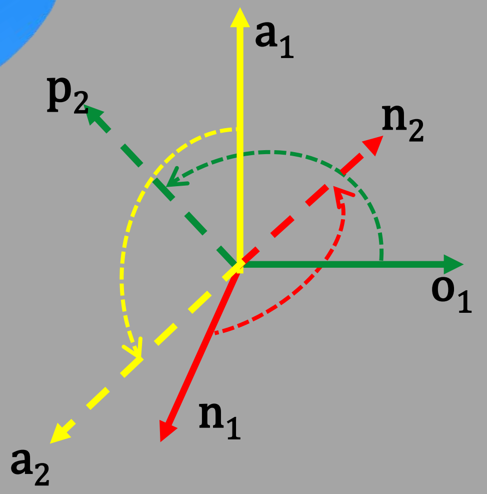
Transfer matrix



$(\{n\}, \{o\}, \{a\}, \{p\}) \in R^{3X1}$

$$T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \{n\} & \{o\} & \{a\} & \{p\} \\ & 0_{1\times3} & & 1 \end{bmatrix}_{4\times4}$$

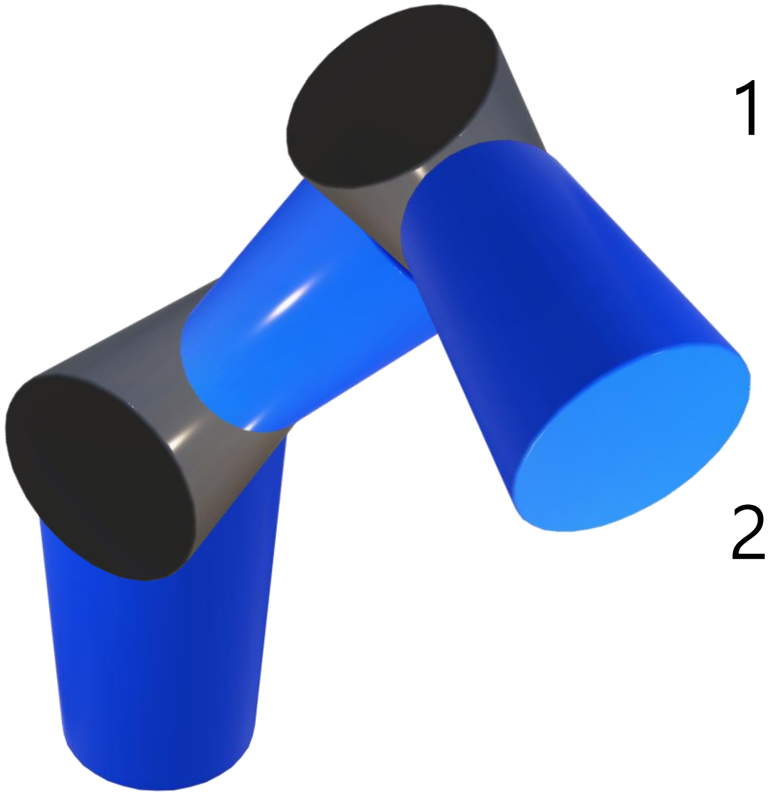
$$T_1 = \begin{bmatrix} R_1 & P_1 \\ 0 & 1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} R_1 & P_1 \\ 0 & 1 \end{bmatrix}$$



$$T_1 T_X = T_2$$

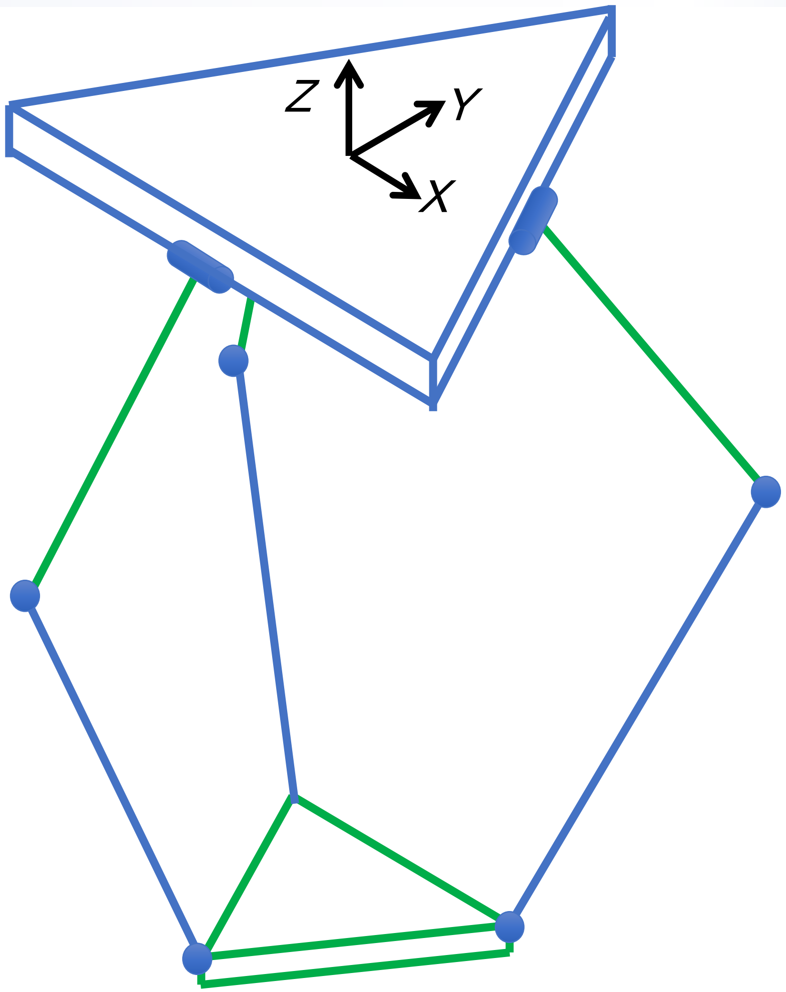
$$T_X = \begin{bmatrix} R_X & P_X \\ 0 & 1 \end{bmatrix}$$

Forward Kinematics – Serial robot



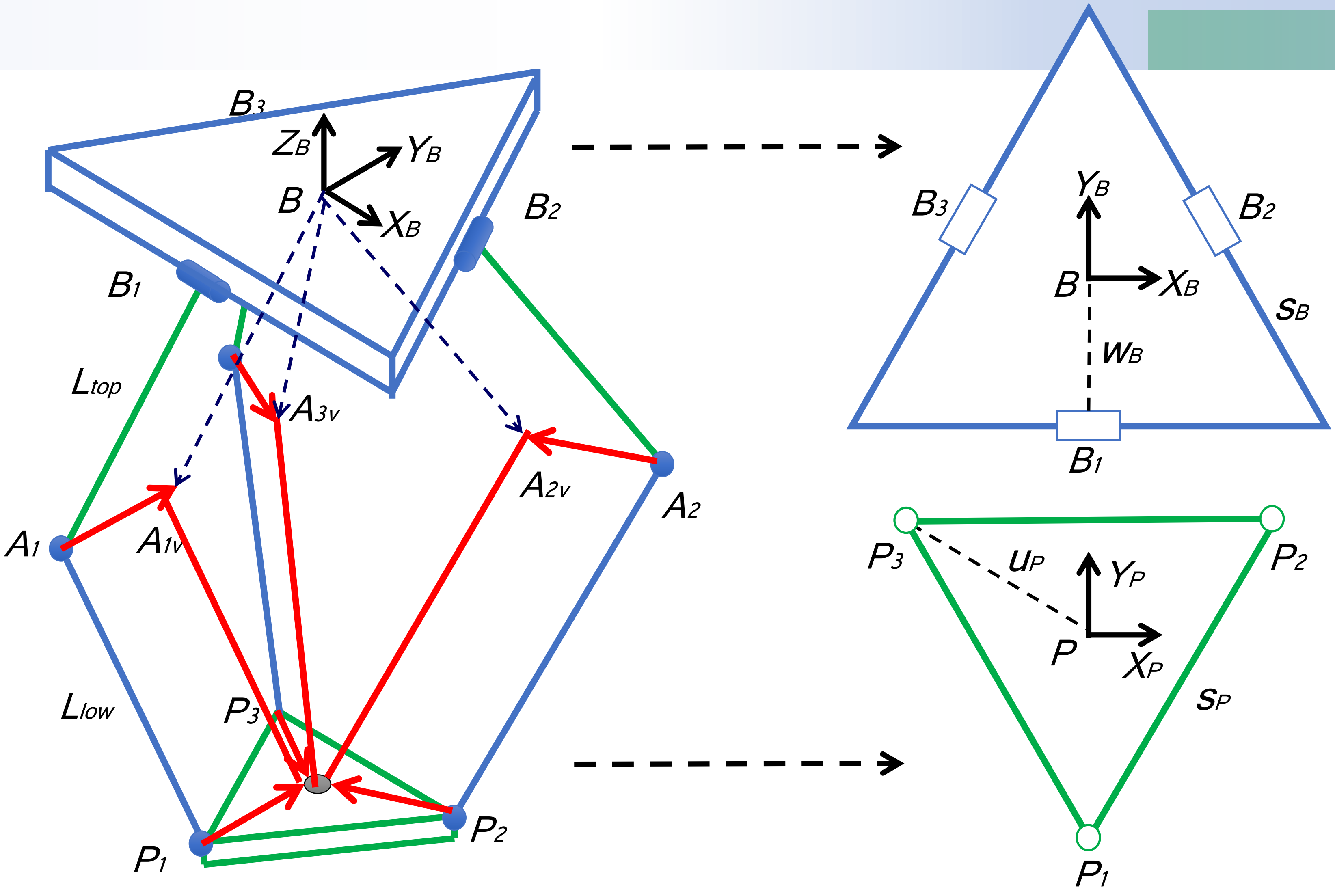
1. Serial robot?
   - The manipulator where axes are linked directly to adjacent axes.
   - Easy to calculate forward kinematics
   - Hard to calculate inverse kinematics
2. So how do we calculate the forward kinematics?
   - We will discuss with D-H parameter later
     - ✓ Orthodox and practical method to express the configuration of the manipulator

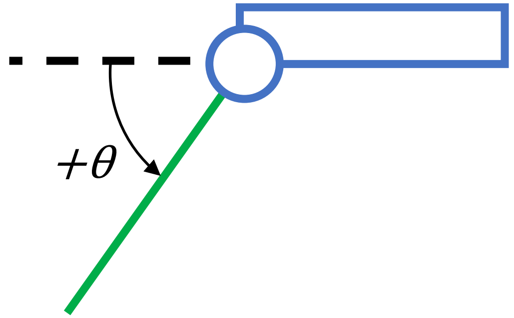
Forward Kinematics – Parallel robot



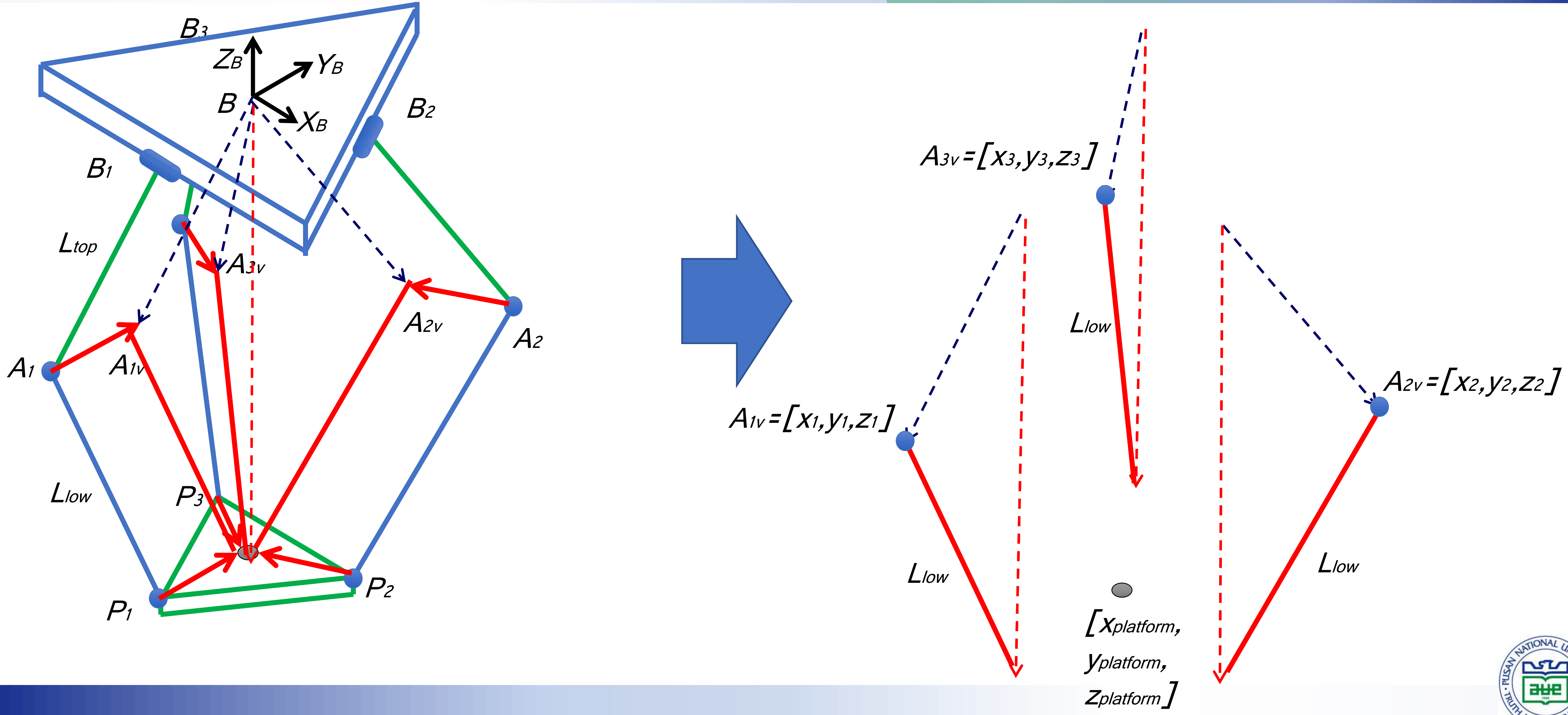
1. Parallel robot?
   - The manipulator where the platform(end-effector) are linked more than 2 axes.
   - Hard to calculate forward kinematics
   - Easy to calculate inverse kinematics
   - For example:
     - ✓ DELTA robot (as shown left)

Forward Kinematics – DELTA 3 robot

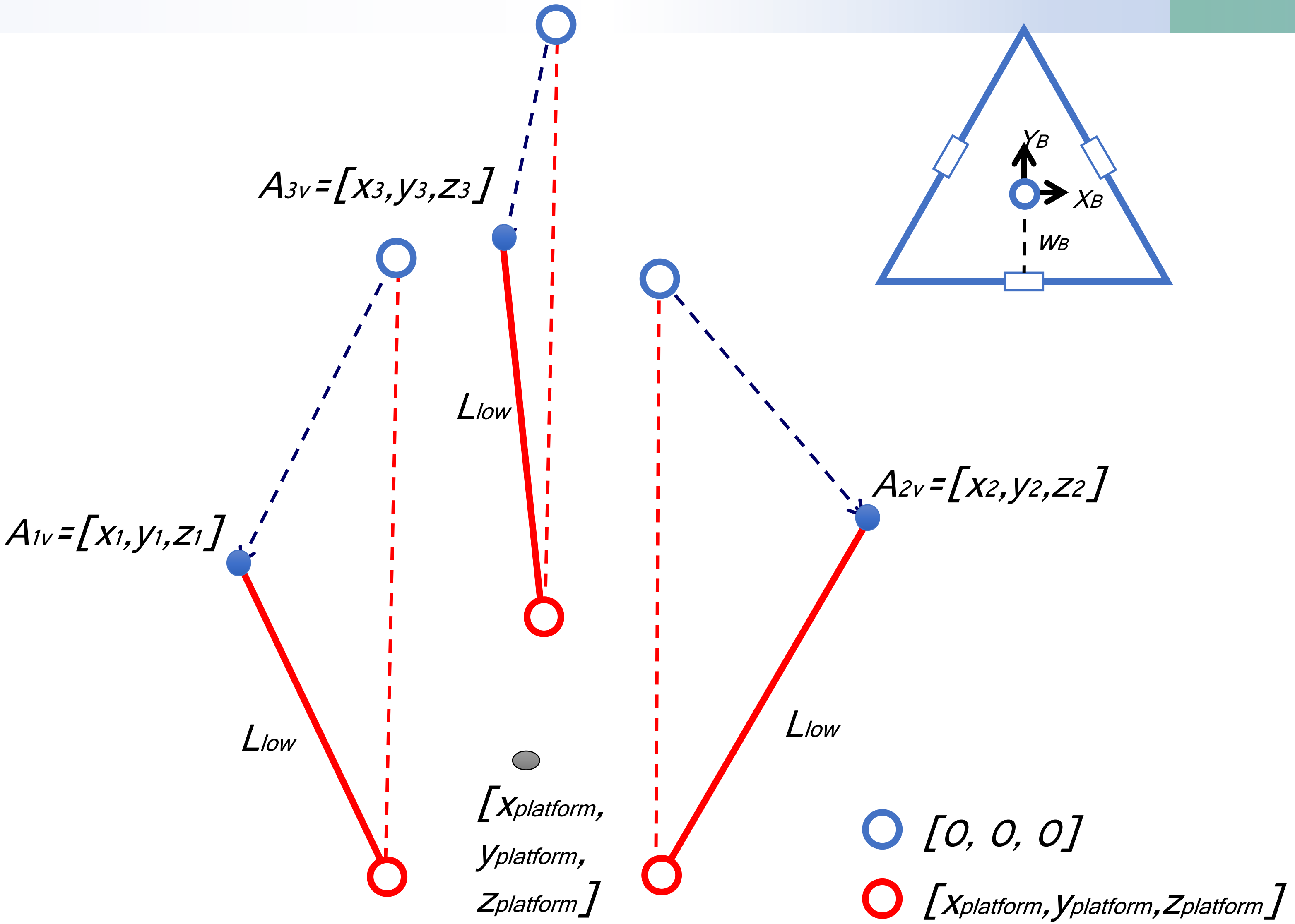




Forward Kinematics – DELTA 3 robot



Forward Kinematics – DELTA 3 robot



$[x_1,y_1,z_1]$, $[x_2,y_2,z_2]$, $[x_3,y_3,z_3]$
are made of theta1, theta2, theta3

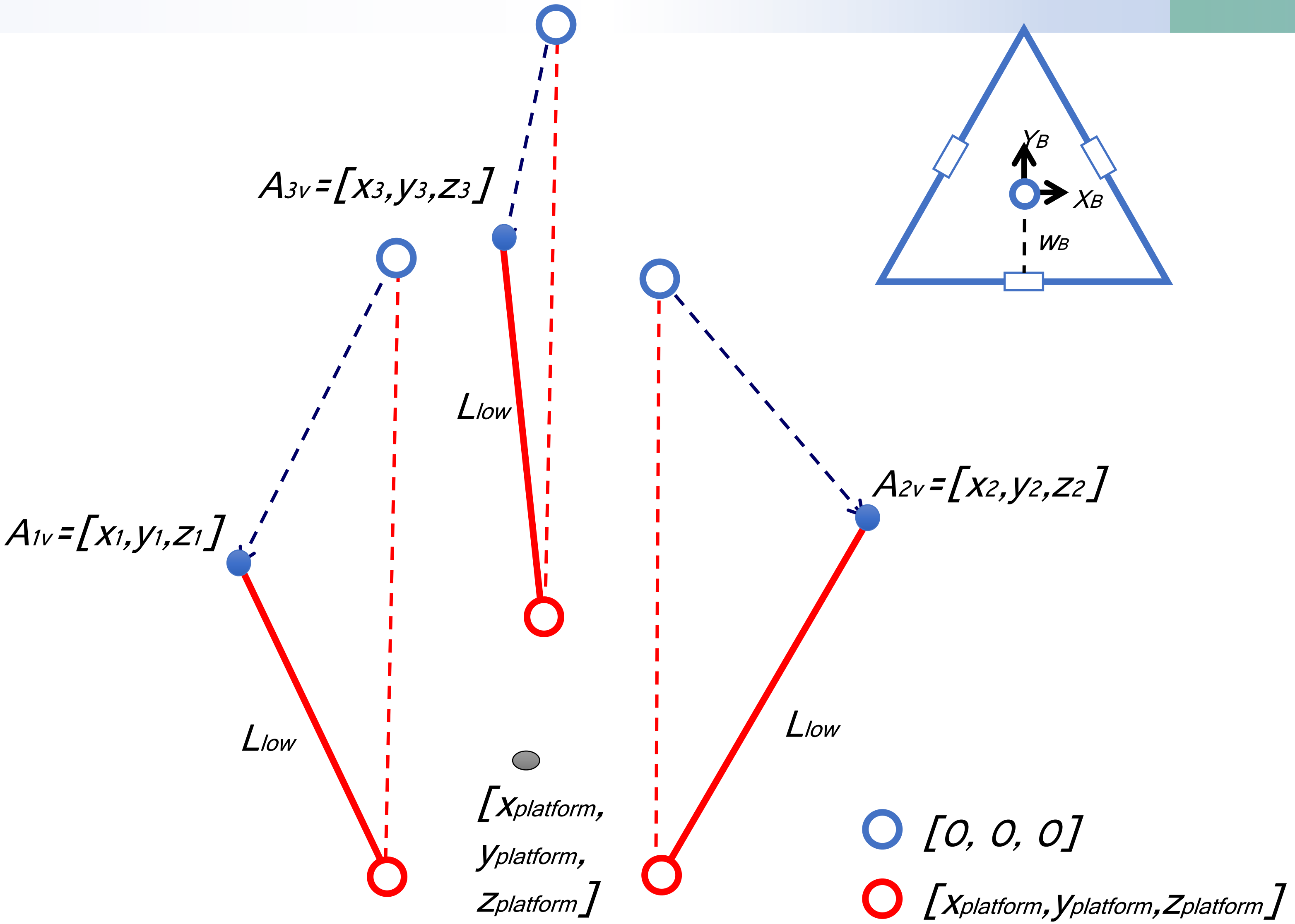
$| [x_{platform},y_{platform},z_{platform}] - [x_1,y_1,z_1] | = L_{low}$
$| [x_{platform},y_{platform},z_{platform}] - [x_2,y_2,z_2] | = L_{low}$
$| [x_{platform},y_{platform},z_{platform}] - [x_3,y_3,z_3] | = L_{low}$

By solving the three equations above,
$[x_{platform}, y_{platfrom}, z_{platform}]$ can be calculated.

$$t = w_B - u_P$$

$$x_1 = 0$$
$$y_1 = -(t + L_{top} \times \cos(theta1))$$
$$z_1 = L_{top} \times \sin(theta1)$$
$$x_2 = (t + L_{top} \times \cos(theta2)) \times \sin(120°)$$
$$y_2 = -(t + L_{top} \times \cos(theta2)) \times \cos(120°)$$
$$z_2 = L_{top} \times \sin(theta2)$$
$$x_3 = (t + L_{top} \times \cos(theta3)) \times \sin(-120°)$$
$$y_3 = -(t + L_{top} \times \cos(theta3)) \times \cos(-120°)$$
$$z_3 = L_{top} \times \sin(theta2)$$

Forward Kinematics – DELTA 3 robot



$$w_1 = x_1^2 + y_1^2 + z_1^2, (x_1 = 0)$$
$$w_2 = x_2^2 + y_2^2 + z_2^2$$
$$w_3 = x_3^2 + y_3^2 + z_3^2$$
$$d_{nm} = (y_2 - y_1) \times x_3 - (y_3 - y_1) \times x_2$$
$$a_1 = (z_2 - z_1) \times (y_3 - y_1) - (z_3 - z_1) \times (y_2 - y_1)$$
$$b_1 = -\frac{\left((w_2 - w_1) \times (y_3 - y_1) - (w_3 - w_1) \times (y_2 - y_1)\right)}{2}$$

$$a_2 = -(z_2 - z_1) \times x_3 + (z_3 - z_1) \times x_2$$
$$b_2 = \frac{\left((w_2 - w_1) \times x_3 - (w_3 - w_1) \times x_2\right)}{2}$$

$$d = b^2 - 4 \times a \times c$$
$$a = a_1^2 + a_2^2 + {d_{nm}}^2$$
$$b = 2 \times \left(a_1 \times b_1 + a_2 \times (b_2 - y_2 \times d_{nm}) - z_1 \times {d_{nm}}^2\right)$$
$$c = (b_2 - y_1 \times d_{nm})^2 + b_1^2 + {d_{nm}}^2 \times \left(z_1^2 - L_{low}^2\right)$$

$$x_{platform} = \frac{a_1 \times z_{platform} + b_1}{d}$$
$$y_{platform} = \frac{a_2 \times z_{platform} + b_2}{d}$$
$$z_{platform} = -\frac{b + \sqrt{d}}{2 \times a}$$