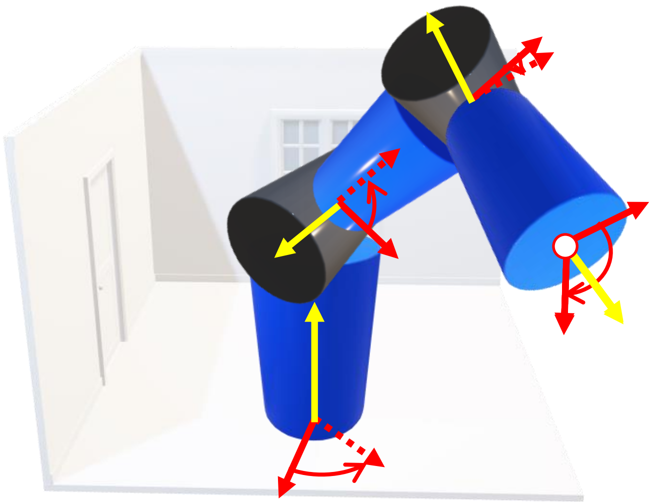
Inverse Kinematics

#4

Inverse Kinematics

1. Inverse Kinematics?
   - From the posture of the end-effector, calculate the corresponding angles
     - ✓ Conventional Inverse Kinematics
     - ✓ Numerical method

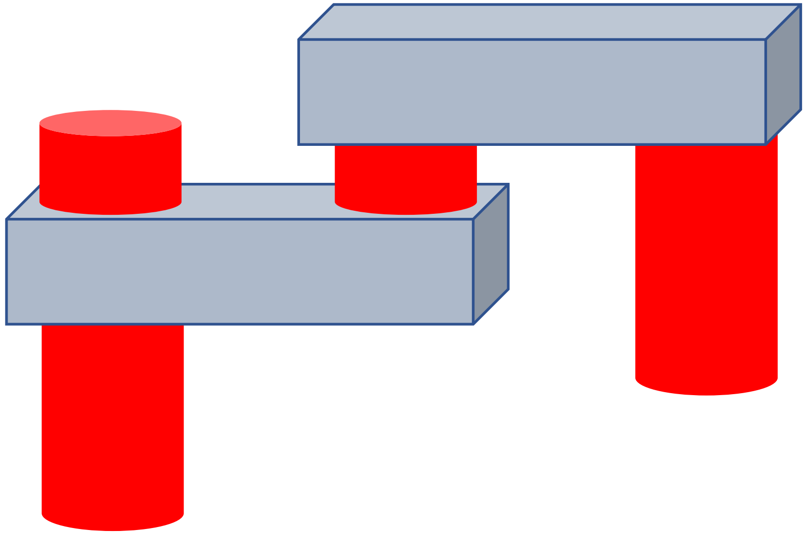


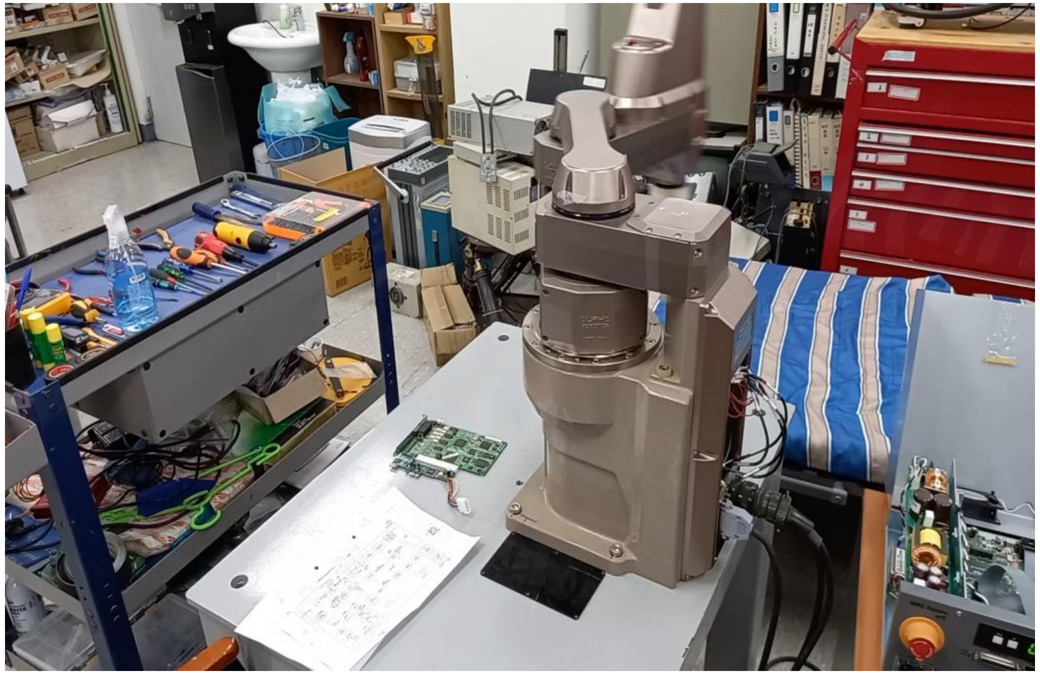
Conventional Inverse Kinematics

2. Conventional Inverse Kinematics
   - Manipulator should be non-redundant
   - The variables are an array of the axis angles.
   - Redundancy can be occurred even if degree of freedom is less than 3 or 6
     - ✓ Snake robot
   - In the conventional manipulator, the set of the answers are usually more than 2.

Example : SCARA robot

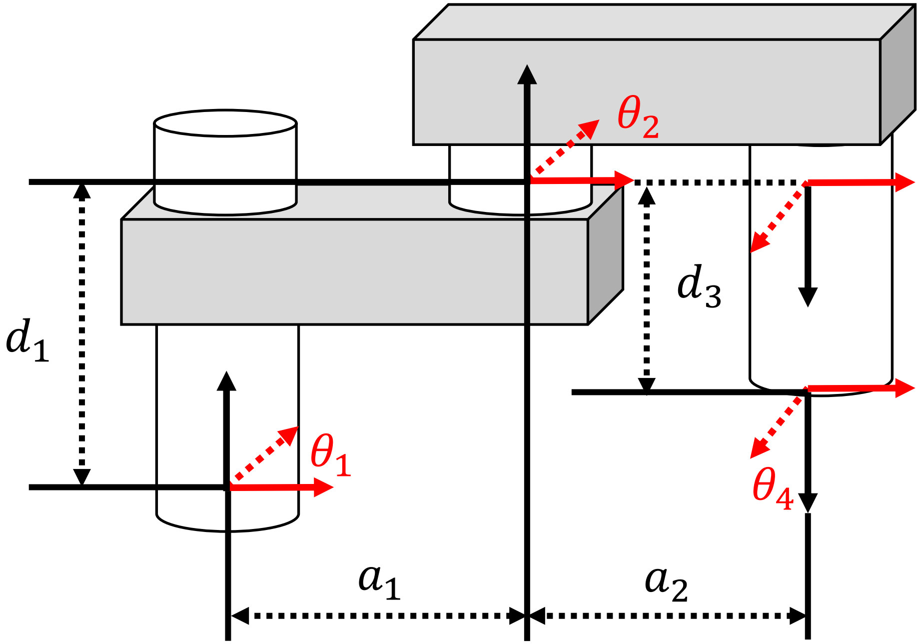
- SCARA robot
  - SCARA?
    - ✓ Selective Compliant Assembly Robot Arm





Example : SCARA robot

- SCARA robot
  - R – R – T – R (4 degree of freedom)
    - ✓ R: rotation, T: translation



Axis	θ	d	a	α
1	θ_1	d_1	a_1	0°
2	θ_2	0	a_2	180°
3	0°	d_3	0	0°
4	θ_4	0	0	0°

Example : SCARA robot

- SCARA robot
  - R – R – T – R (4 degree of freedom)
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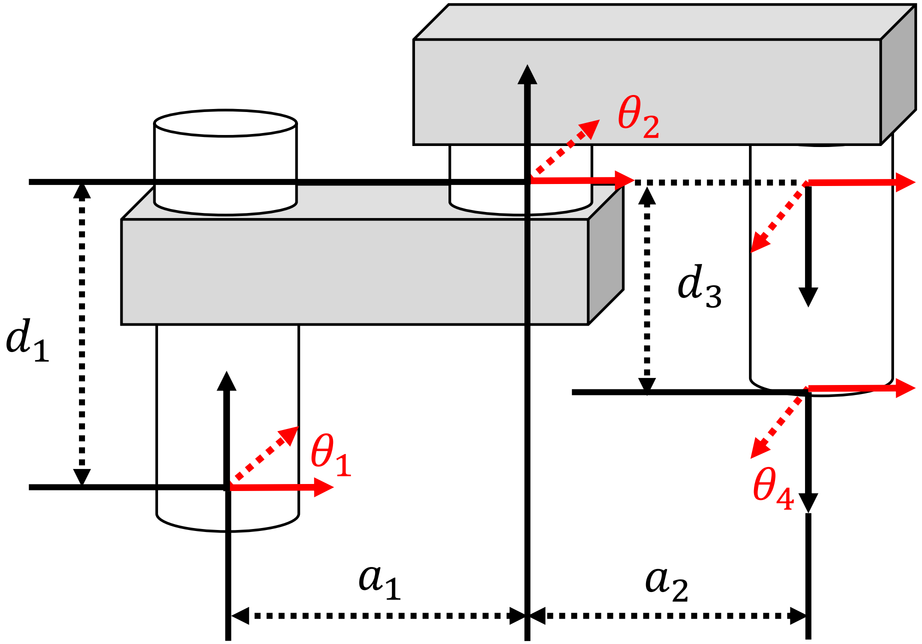
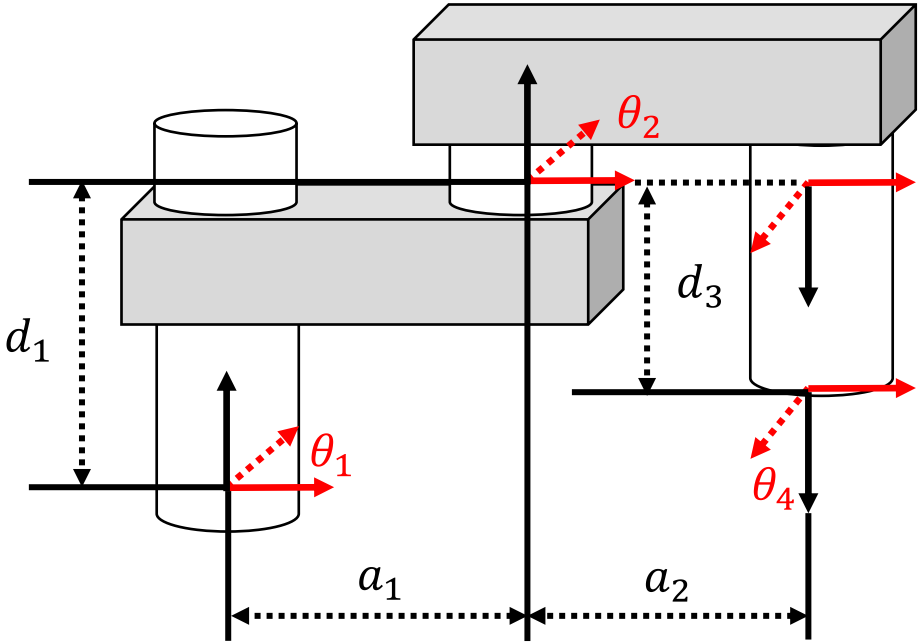


Axis	Equation
T_1	$\left[\begin{array}{cccc} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{array}\right]\left[\begin{array}{cccc} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{array}\right]$
T_2	$\left[\begin{array}{cccc} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{array}\right]\left[\begin{array}{cccc} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{array}\right]\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{array}\right]$
T_3	$\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{array}\right]$
T_4	$\left[\begin{array}{cccc} \cos(\theta_4) & -\sin(\theta_4) & 0 & 0 \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{array}\right]$

Example : SCARA robot

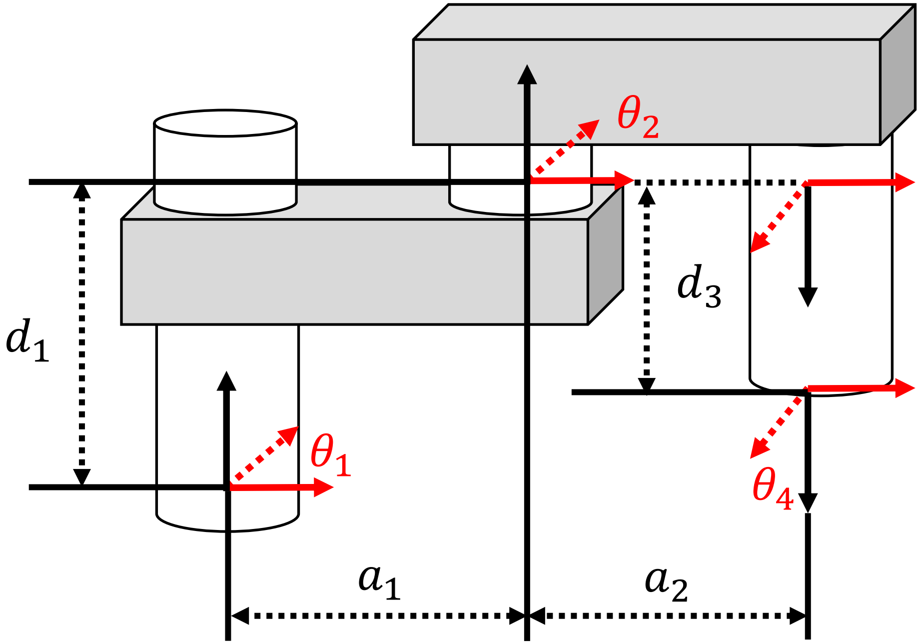
- SCARA robot
  - R – R – T – R (4 degree of freedom)
    - ✓ R: rotation, T: translation



Axis	Equation
T_1	$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & a_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & a_1\sin(\theta_1) \\ 0 & 0 & 1 & d_1 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{bmatrix}$
T_2	$\begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 & a_2\cos(\theta_2) \\ \sin(\theta_2) & -\cos(\theta_2) & 0 & a_2\sin(\theta_2) \\ 0 & 0 & -1 & 0 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{bmatrix}$
T_3	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{bmatrix}$
T_4	$\begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & 0 \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \multicolumn{3}{c}{\mathbf{0}} & 1 \end{bmatrix}$

Example : SCARA robot

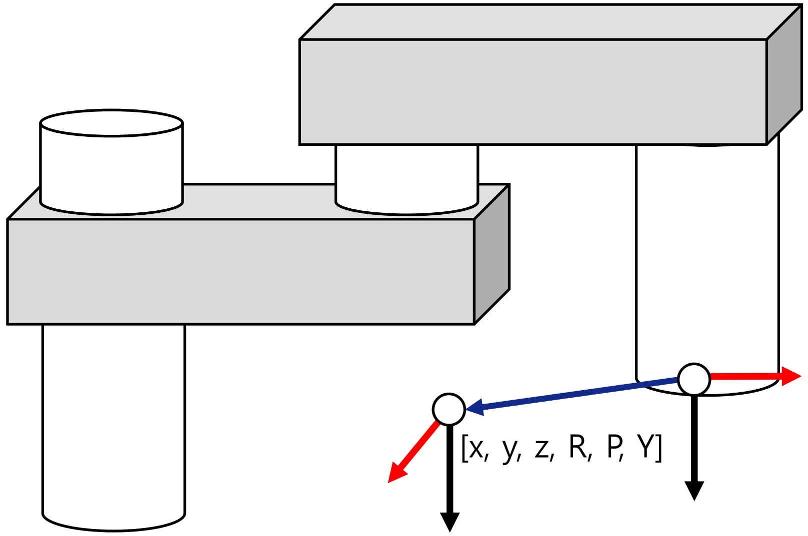
- SCARA robot
  - R – R – T – R (4 degree of freedom)
    - ✓ R: rotation, T: translation



$$T_1T_2T_3T_4$$
$$= \begin{bmatrix} \cos(\theta_1+\theta_2-\theta_4) & \sin(\theta_1+\theta_2-\theta_4) & 0 & a_1\cos(\theta_1)+a_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2-\theta_4) & -\cos(\theta_1+\theta_2-\theta_4) & 0 & a_1\sin(\theta_1)+a_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & d_1-d_3 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

Example : SCARA robot

- SCARA robot
  - Reference posture : [x, y, z, R, P, Y]
    - ✓ [(Cartesian coordinate), (Euler Representation)]



$$T_1T_2T_3T_4$$

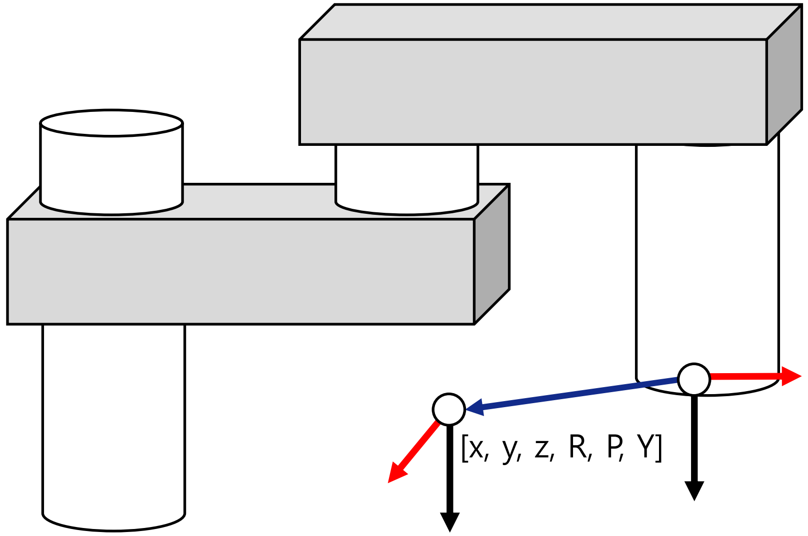
$$= \begin{bmatrix} \cos(\theta_1+\theta_2-\theta_4) & \sin(\theta_1+\theta_2-\theta_4) & 0 & a_1\cos(\theta_1)+a_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2-\theta_4) & -\cos(\theta_1+\theta_2-\theta_4) & 0 & a_1\sin(\theta_1)+a_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & d_1-d_3 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\mathrm{P})\cos(\mathrm{Y}) & -\cos(\mathrm{P})\sin(\mathrm{Y}) & \sin(\mathrm{P}) & x \\ \cos(\mathrm{R})\sin(\mathrm{Y})+\cos(\mathrm{Y})\sin(\mathrm{P})\sin(\mathrm{R}) & \cos(\mathrm{R})\cos(\mathrm{Y})-\sin(\mathrm{P})\sin(\mathrm{R})\sin(\mathrm{Y}) & -\cos(\mathrm{P})\sin(\mathrm{R}) & y \\ \sin(\mathrm{R})\sin(\mathrm{Y})-\cos(\mathrm{R})\cos(\mathrm{Y})\sin(\mathrm{P}) & \cos(\mathrm{Y})\sin(\mathrm{R})+\cos(\mathrm{R})\sin(\mathrm{P})\sin(\mathrm{Y}) & \cos(\mathrm{P})\cos(\mathrm{R}) & z \\ & \mathbf{0} & & 1 \end{bmatrix}$$

In this equation we can know that the R and P is restricted to 180 and 0 degree, respectively.

Example : SCARA robot

- SCARA robot
  - Reference posture : [x, y, z, R, P, Y]
    - ✓ [(Cartesian coordinate), (Euler Representation)]



$$T_1T_2T_3T_4$$

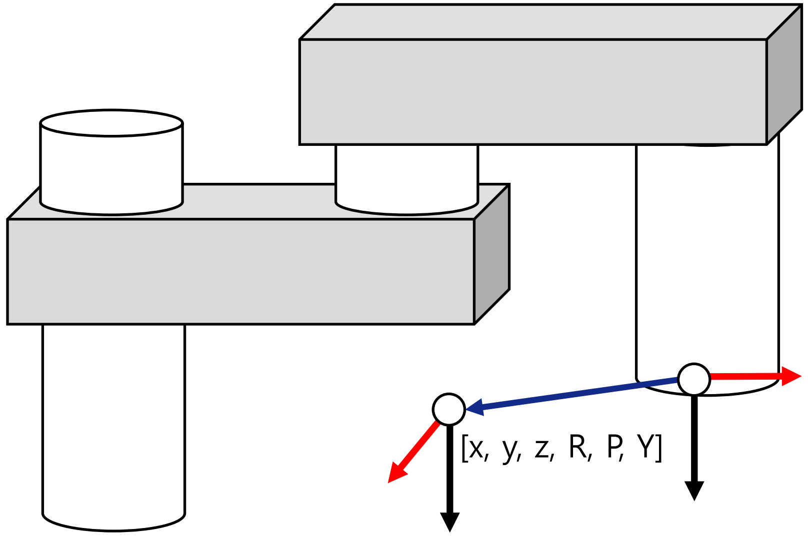
$$= \begin{bmatrix} \cos(\theta_1+\theta_2-\theta_4) & \sin(\theta_1+\theta_2-\theta_4) & 0 & a_1\cos(\theta_1)+a_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2-\theta_4) & -\cos(\theta_1+\theta_2-\theta_4) & 0 & a_1\sin(\theta_1)+a_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & d_1-d_3 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\mathrm{Y}) & -\sin(\mathrm{Y}) & 0 & x \\ -\sin(\mathrm{Y}) & -\cos(\mathrm{Y}) & 0 & y \\ 0) & 0 & -1 & z \\ & \mathbf{0} & & 1 \end{bmatrix}$$

In this equation we can know that the R and P is restricted to 180 and 0 degree, respectively. (Characteristics of the SCARA Robot)

Example : SCARA robot

- SCARA robot
  - Reference posture : [x, y, z, R, P, Y]
    - ✓ [(Cartesian coordinate), (Euler Representation)]

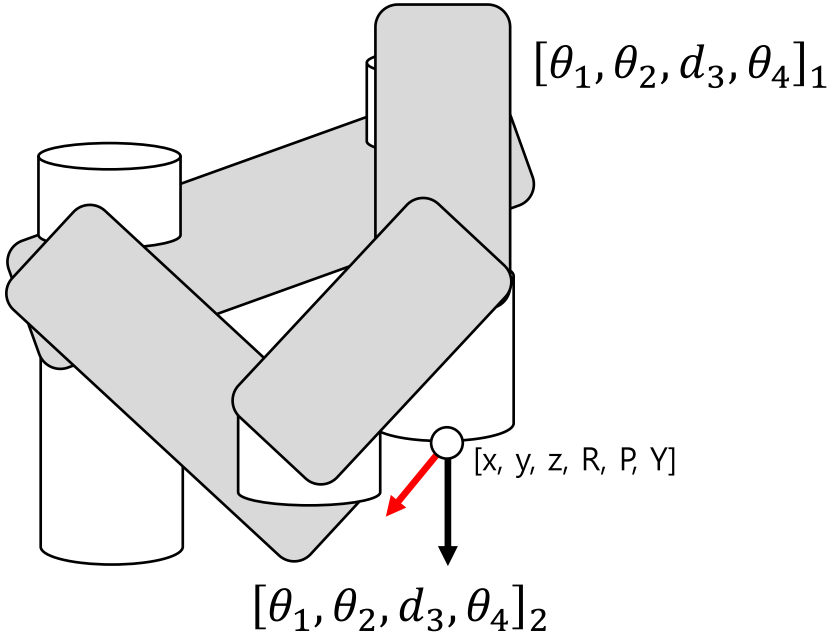


$$\tan(\theta_1 + \theta_2 - \theta_4) = -\tan(\mathrm{Y})$$
$$a_1 \cos(\theta_1) + a_2 \cos(\theta_1 + \theta_2) = x$$
$$a_1 \sin(\theta_1) + a_2 \sin(\theta_1 + \theta_2) = y$$
$$d_1 - d_3 = z$$

In this equation we can know that the R and P is restricted to 180 and 0 degree, respectively. (Characteristics of the SCARA Robot)

Example : SCARA robot

- SCARA robot
  - By characteristics of the trigonometrical function, we have two sets of answers.



$$\tan(\theta_1 + \theta_2 - \theta_4) = -\tan(\mathrm{Y})$$
$$a_1 \cos(\theta_1) + a_2 \cos(\theta_1 + \theta_2) = x$$
$$a_1 \sin(\theta_1) + a_2 \sin(\theta_1 + \theta_2) = y$$
$$d_1 - d_3 = z$$

Set of answer : $\{[\theta_1, \theta_2, d_3, \theta_4]_1, [\theta_1, \theta_2, d_3, \theta_4]_2\}$

Example : Articulated robot

- Articulated robot
  - The serial robot, which resembles with a human arm.
  - What if the robot arm has redundancy?
    - ✓ When the robot arm has an infinite set of answers
  - This can't be calculated with conventional inverse kinematics
    - ✓ For this situation, we need to apply numerical method solution